Math 110/110S
Introduction to Statistics
With Support
Instructors' Manual
Version 1

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Overview

This manual contains facilitation notes for the Math 110/110S activity packet.

The Math 110/110S activity packet dovetails with course materials available to students for free in Canvas. The Canvas materials were adapted from the Open Learning Initiative's Concepts of Statistics course. There is no textbook. These materials require the use of StatCrunch.

Structure of the facilitation notes

The facilitation notes for each activity are divided into three parts:

- Set-up: (Before group work) suggestions for introducing new concepts before students work in groups. The Set-up is often a short interactive class discussion, but it can also include mini-lecture
- Implementation: (During group work) suggestions for monitoring learning and trouble-shooting student difficulty during group work
- Closure: (After group work) suggestions for checking for understanding and making connections between concepts

Additional instructor resources are available in Canvas in the section titled Math 110 Instructors' Resources, including a template syllabus, group quizzes and exams.

Suggestions for using these notes

For some activities the facilitation notes are long and detailed. They will be difficult for you to digest while you are on your feet in class.

We suggest that you read the activity first in the student activity packet, then read the facilitation notes. Finally, make a short summary of the facilitation notes or annotate them for quick reference during class.

Of course, you may have other ideas for facilitating the activities, but keep your eye on the clock. It is natural to get worried about pacing and resort to lecture to "cover" more material quicker. However, lecture does not equal learning. The facilitation notes map out a strategy for getting through all of the material in Math 110/110S using an interactive pedagogy designed to deepen students' learning.

Cycle of learning in this student-centered Math 110/110S course

Class activities: Making sense of new ideas

Students work through an activity or series of activities to grapple with new concepts.

Instructor's role:

- Set up the activity to prepare students for productive struggle that fosters deeper learning of concepts and connections between concepts.
- Monitor and strategically intervene during group work to help students make sense of concepts.
- Conclude the lesson to ensure that students have achieved the learning goal.
- Record participation in the activity. Class activities comprise 10% of the course grade.

<u>Canvas: Solidifying new ideas</u>

Students read concise explanations and examples of the ideas discussed in class. Embedded in the reading are numerous low-stakes quizzes and some discussions that help students assess their understanding. Feedback is automatic and students have opportunities make corrections.

Some Canvas exercises require StatCrunch. Full StatCrunch instructions are embedded in Canvas in a just-in-time way.

Instructor's role:

- Monitor student work in Canvas by glancing through the Canvas gradebook before a due date. Use Canvas gradebook to send out encouraging emails to motivate students and keep them on track.
- Assign online groups for each module, set due dates for quizzes and Canvas discussions, grade about 2 discussions a week. All other Canvas assessments are automatically graded and entered into the Canvas gradebook.

<u>Labs, Group Projects, Group Quizzes: Pulling it altogether</u>

At the end of a Unit, students participate in a Group Project in class and also complete a written lab assignment. These culminating activities and assignments require students to synthesize concepts in the context of a real-life application.

At the end of a long Unit or a series of shorter Units, students also participate in a Group Quiz to help them prepare for exams.

Instructor's role:

- Group Projects: provide materials for making posters, monitor and strategically intervene if necessary during group work, facilitate the gallery walk, led the discussion after the gallery walk. Record participation grade.
- Group quizzes: grade one paper for each group.
- Labs: grade written labs.

Math 110/110S Pace Chart

Math 110/110S is a 6-unit course combination, with 8 hours of in-class contact. This pace chart assumes 31 class meetings @ 125-minutes per class. Each day's plans account for up to 200 minutes of instruction so that you will have time for several breaks. Out of 16 weeks of instruction, the pace chart includes time for the final exam and final project presentations; in addition, one day is set aside for a holiday. The holiday is arbitrarily placed into Week 14.

Notes:

1) This pace chart lays out a plan to achieve all Math 110 and Math 110S learning goals and addresses all required content topics.
2) If homework says "Mod 5", this means all pages associated with Module 5 in Canvas, <u>including the Module Checkpoint (Ckpt) quiz</u>.
3) Extensive StatCrunch instructions with pictures are embedded in Canvas when needed for an exercise.
4) See the instructors' manual for extensive facilitation notes for these activities.

Week	Day 1 Activities	Day 2 Activities
1 1st Poster Session, Activities to set norms and clarify expectations Start Unit 2 Distributions Quantitative Data	Welcome, introduce yourself (15 mins) Ice breakers (35 mins) Mod 1 Cereal Activity (125 mins) Setting Group Norms (25 mins) Canvas homework preview (5 mins) –1st HW includes a discussion post. Pull up that assignment in Canvas, discuss general directions. HW: Read the syllabus, complete Mods 1-3 in Canvas, purchase a StatCrunch license and set up StatCrunch account	Syllabus quiz (15 mins) 4.1 (75 mins) 4.2 (90 mins) High Quality Feedback (15 mins) Canvas HW preview (5 mins) HW: Feedback on DB* Mod 2 (2 of 4 discussion 1), Part of Mod 4 – beginning through Dotplots (2c of 2 quiz 5), * DB=discussion board

2 Unit 2	4.3 (100 mins) 4.4 Lab (60 mins) How Learning Happens (15) HW preview (15 mins)– Canvas Histogram Lab requires StatCrunch (demo making a histogram to reassure students; Canvas has instructions) HW: Finish Mod 4 (including Ckpt), Feedback on DB Mod 4–Dotplots (2b of 2 discussion 1) 4.4 Lab (if not finished in class)	5.1 (90 mins) 5.2 (70 mins) Mod 2 Activity in packet (30 mins) HW preview: Students will do all of Mod 5, even though we have not completed the Mod 5 activities in class. HW: Mod 5 (including Ckpt), Feedback on DB Mod 4-Histogram Lab (discussion 2) AND Histograms (4 of 4 discussion 3), Read Brainology in Mod 1 of packet
3 Unit 2	5.3 (70 mins) 6.1 (70 mins) Discuss Brainology (50 mins) HW: Feedback on DB Mod 5-Reflections on Success (discussion 1), Mod 6 all, Read "You Can Grow Your Brain"	6.2 (80 mins) 7.1 (100 mins) Discuss "You Can Grow Your Brain" (25 mins) HW: Feedback on DB Mod 6 (2c of 3 discussion 1) 6.3 Lab
4 Finish Unit 2 Unit 3 Linear Regression	7.2 (65 mins) 7.3 (60 mins) 7.4 (60 mins) 7.5 Mod 7 Lab (Get a start: 15 mins) HW: Mod 7 all, 7.5 Lab	7.7 Unit 2 Project (70 mins) Group quiz Unit 2 (40 mins) – Use 7.6 Unit 6 Summary Lab as a quiz 8.1 (80 mins) HW: Feedback on DB Mod 7-Drinking Habits Lab (discussion 1), Mod 8 Part 1 through Scatterplots (4 of 4)

5 Finish Unit 3 Unit 4 Relationships Categorical Variables	8.2 (80 mins) 8.3 (35 mins) 8.4 (70 mins) HW: Mod 8 finish (including Ckpt)	9.1 (65 mins) 9.2 (50 mins) Assessing the Fit of a Line (15 mins) 9.4 Unit 3 Group Project (70 mins) HW: Mod 9 all, Checkpoint Unit 3, 9.3 Unit 3 Lab
6 Exam 1 Units 2-4 Start Unit 5 Probability and Probability Distributions	10.1 (50 mins) 10.2 (50 mins) 10.4 Unit 4 Project (60 mins) 10.3 Unit 4 Lab (40) HW: Mod 10 all, Unit 4 Checkpoint,	Group quiz Units 2-4 (60 mins) Interactive exam prep (80 mins)– Teach a Problem or something similar Discuss blog “This is Why You Should Be Proud of Making Mistakes” (30 mins) HW: Feedback on DB Mod 10 Treating Depression Lab (discussion 1&2), Study for Exam
7 Finish Unit 5 Start Unit 6 Statistical Studies	EXAM (120 mins) Collect student exams. Turn the exam into a learning opportunity by doing parts of the exam interactively. (80 mins) To prevent exam work from leaving the class, distribute colored scratch paper. Students turn in their scratch paper before leaving. Project one problem at a time. Think-Square-Share, with student presentations.	11.1 (30 mins) 11.2 (50 mins) 12.1 (60 mins) 12.2 (60 mins) HW: Mod 11 all, Mod 12 through Mod 12-Intro to Normal Probability Distributions (2c of 5 quiz 6)

8 Finish Unit 6 Start Unit 7 Linking Probability to Statistical Inference	12.3 (45 mins) 12.4 Lab (40 mins) Mod 13 (45 mins) 14.1 (35 mins) 14.2 SKIP 15.1 (35 mins) HW: Feedback on DB Mod 11 Probability Applets Lab (discussion 1), Finish Mod 12, Unit 5 Checkpoint, Mod 13 all (short), Mod 14 all (short)	15.3 Unit 6 Group Project (100 mins) 15.2 Unit 6 Lab (40 mins) Mod 16 Activity (10 mins) 17.1 (50 mins) HW: Feedback DB Mod 13 (1a of 4 discussion 1), Mod 15 all, Unit 6 Checkpoint, Mod 16 all,
9 Unit 7	Discuss Canvas Intro to Inference (2 of 2) (20 mins) 17.2 (55 mins) 17.3 (60 mins) 17.4 (60 mins) HW: 15.2 Lab Feedback DB Mod 15 (1 of 2 discussion 1) & (2 of 2 discussion 2), Mod 17 all	Group quiz Mod 17 (40 mins) 18.1 (50 mins) 18.2 (45 mins) 18.3 (50 mins) HW: Feedback DB Mod 17 (3 of 6 discussion 1), (5 of 6 discussion 2), (6 of 6 discussion 3), Mod 18 through Intro to Statistical Inference (2b of 3 quiz 4)
10 Finish Unit 7 Start Unit 8 Proportions	18.4 (55 mins) 18.5 Lab (45 mins, challenging!) Group quiz Mod 18 (20) 19.1 (80 mins) HW: Feedback DB Mod 18 (2b of 3 discussion 1), Finish Mod 18, Unit 7 Checkpoint, Mod 19 all, Finish 18.5 Lab	20.1 (80 mins) 20.2 (15 mins) 20.3 (80 mins)–includes extra practice on interpreting P-value. See suggestions in facilitation notes. HW: Feedback DB Mod 19 (2 of 3 discussion 1), Mod 20 all

11 Finish Unit 8 Proportions	21.1 (80 mins) 21.2 (60 mins) 21.3 Lab (50 mins) HW: Feedback DB Mod 20 (4a of 5 discussion 1), Mod 21 all, Unit 8 Checkpoint	21.4 Unit 8 Project (70 mins) Group Quiz Units 5-8 (60 min) Exam prep: speed-dating, Teach a Problem (60 mins) HW: Feedback Mod 21 Cell Phone Lab (discussion 1), Study for exam
12 Exam 2 Units 5-8 Start Unit 9 Means	EXAM Units 5-8 (120 min) Collect student exams. Turn the exam into a learning opportunity by doing parts of the exam interactively. (80 mins) To prevent exam work from leaving the class, distribute colored scratch paper. Students turn in their scratch paper before leaving. Project one problem at a time. Think-Square-Share, with student presentations.	22.1 (70 mins) 22.2 (60 mins) Start work on final project (70 min) Consider randomly assign groups so that no one is left out. Groups pick a topic, write a research question (review components of a good research question–Mod 13) and design their study (similar to Unit 6 Project) HW: Mod 22 all
13 Unit 9 Means	23.1 (65 mins) 23.2 (65 mins) Continue work on final projects (70 mins): refine study design, clarify variables and variable type. Make plans for collecting data. Poster & gallery walk? HW: Mod 23 all, Start data collection for final project	24.1 (60 mins) 24.2 (55 mins) 25.1 (90 mins) HW: Mod 24 all Continue data collection for final project.

14 Finish Unit 9 Means Unit 10 ANOVA	25.3 Unit 9 Project (70 mins) 25.2 Lab (challenging!) (65 mins) Final project work (50 mins): do a dry run with fictitious data in StatCrunch. Make a spreadsheet, create graphs and find numerical summaries, conduct statistical tests. HW: Feedback DB Mod 24 (4c of 5 discussion 1), (4d of 5 discussion 2), (5a of 5 discussion 3), Mod 25 all, Checkpoint Unit 9, Finish data collection for final project	Mod 26 (110 mins) Work on final projects (90 mins): Enter and analyze the real data in StatCrunch. Write drafts of each section of the report. HW: Feedback DB Mod 25 (discussion 1), Mod 26 all, Unit 10 Checkpoint
15 Unit 11 Chi-square	Mod 27.1 (100 mins) SKIP 27.2 and 27.3 Group Quiz Units 9-11 (50 mins) Finalize project reports and do dry run of project presentation (50 mins) HW: Feedback DB Mod 26 (6 of 7 discussion 1), Mod 27 all, Unit 11 Checkpoint, Finish projects	PLACEHOLDER FOR HOLIDAY
16 Final Exam and Final Project	Project presentations (120 mins) Group quiz–comprehensive or Teach A Problem (80 mins) HW: Study for Final Exam	FINAL EXAM

Tips for Facilitating Group Work

We have five strategies that pre-empt a lot of common problems with group work:

1) Provide a compelling rationale for teamwork
2) Collaboratively develop group norms
3) Use random assignment of groups
4) Hold students accountable
5) Be positive and encouraging

Five strategies for effective group work

1) Provide a compelling rationale for teamwork

Students who do not understand why we use group work may complain that you are forcing them to teach themselves and that you aren't doing your job. Some may not buy into working with others, "I learn better on my own."

Activities in facilitation notes and/or the packet : How Learning Happens, Module 2 Learning Strategies

2) Collaboratively develop group norms

Students complain about working in groups for many reasons. It is important to listen to student fears and experiences about group work, then move forward to set norms for interaction that address their concerns.

Activities in the facilitation notes and/or packet: Setting Group Norms

3) Use random assignment of groups

Random assignment of groups addresses the following issues:

- A few students are domineering or "know-it-alls" and appear to be doing most of the work. Some students are disengaged.
- Cliques of students (e.g. student athletes or high school friends) always sit together and engage in unproductive behaviors.
- Some groups are strong and others are weak. Students in weaker groups are not achieving the learning goals.

Of course, students may also complain about random assignment of groups. It is important that you be consistent in using random assignment every class period. Do not capitulate to complaints by letting some groups remain together, even if these groups are productive.

Random assignment of groups provides all students the opportunity to learn important "soft" skills that are part of working in teams. A student will learn to navigate his/her role in each group. Someone who is dominant in one group may be less so with a different group. Someone who is shy in one group may be more willing to talk in another. Cliques are broken up and new friendships are formed.

Suggestions for how to randomly assign groups:

Whichever method you use to randomly assign groups, make a map on the board so that students know where to sit.

Require students in a group to rearrange their chairs so that they are facing each other. Do not let groups sit in a row.

- Use Canvas. From your Canvas homepage, select the People tab, select "Group Set" option in the top right corner and select the number of groups desired.

  If the default font size is too small for students to see, use the mouse wheel to increase the font size.You will be able to see half of the assigned groups at a time in the largest font. Take two screen shots with the larger font size and project the two screen shots side by side.

- Use playing cards. If you use playing cards, shake up the way you assign groups. If you don't do this, students will trade cards to work with their friends. For example, one day all the 2's, 3's, etc. work together. Another day group by suit, e.g. Ace, 2, 3, 4 of one suit together, etc.

4) Hold students accountable

When students are goofing off or appear to be copying from other students instead of really trying to learn, you need to consistently use strategies that hold individuals accountable.

Suggestions for increasing accountability during group work:

After group work, use the following types of strategies to hold individuals accountable. Keep it upbeat, not punitive. Remind students that "do and say" produces learning that sticks. If they are chosen to present, explain or represent their group, this is the best learning opportunity that the class can give.

- Speed-dating requires every student to be able to share the work of their group.

- Do an Ambassador Exchange and choose disengaged students to be the Ambassadors. Give them a heads up by choosing Ambassadors before the end of group work. Give the group a few minutes to do a dry run to help the disengaged student get ready.

- During a poster session, choose the disengaged student to be the presenter. Give them a heads up by choosing presenters before the end of group work. This provides a low stakes opportunity for the student to repeatedly explain ideas and improve with repetition. Give the group a few minutes to do a dry run to help the disengaged student get ready.

- Do a Whole Class Quiz and call on the disengaged students. During group work, let the class know that you plan to do a Whole Class Quiz and that you will be randomly choosing students to answer. You can use random selection or just pretend to use it and choose students that appeared disengaged to answer questions.

- Establish an expectation for students to present solutions at the board. If students are goofing off, assign them a problem to present while groups are still working. If they struggle at the board, they can ask for hints.

- During class discussion, call on students who appeared to be disengaged. Allow them to pass if they want to but then return to them to answer another question or ask them to paraphrase someone else's answer or tell them to ask a question that will help them be able to answer.

5) Be positive and encouraging

Sometimes goofing off or copying is grounded in fear. Students don't think they can do the work, so they don't try.

In her book The College Fear Factor: How Students and Professors Misunderstand Each Other, Rebecca Cox writes that a high level of student fear is "a dimension of student experience that has emerged in every study I have conducted, across community colleges in different regions of the country and with a highly diverse range of students" (20).

"A significant component of students' stress was directly linked to their doubts about succeeding in college and realizing their career goals. For some students, this fear – a natural part of any life transition – was heightened by their past experiences with failure in academic contexts…" (25).

"Essentially, students were afraid that their professors would irrevocably confirm their academic inadequacy" (26).

Suggestions for addressing the fear factor:

- Praise and micro-affirmations: In your comments and actions, praise effort, risk-taking, improvement, and errors that lead to learning. Use students' names when you praise them. Give kudos to groups that worked well together, grappled with disagreement, utilized feedback, etc.

- Make the connections to learning clear: In your comments and actions, always emphasize the learning benefits of what you are asking students to do.

Other assignments that may help address the fear factor:

- Assign the article "Brainology," which is included in the packet along with discussion questions.

- Assign the article "You Can Grow Your Brain," which is included in the packet. Ask students to explain how this class is structured to grow the brain.

- Assign "You need to make mistakes to get ahead." Share a story of when you learned from a mistake. Acknowledge embarrassment. Reinforce this message by celebrating mistakes as gifts of learning (e.g. make a habit of asking, "Can anyone share a mistake they made today that helped them learn something?) http://www.cnn.com/2011/LIVING/08/02/need.to.make.mistakes/index.html

- Assign the article "When to Let Students Struggle" and have a class discussion of the research cited there and how it relates to students' experiences and the structure of Math 110. (http://ideas.time.com/2012/04/25/why-floundering-is-good/)

Trouble shooting other issues:

Students are sitting and staring at the activity with little conversation.

Sometimes students think problem solving is like waiting for a light bulb to turn on. They expect solutions to appear quickly and in complete form. They may also think that struggle is an indication of being stupid. The goal is to get them to engage and try something.

Suggestions:

- Ask a student in the group to read the problem out loud. Ask other group members: What is being asked? Can they identify some useful information? What seems puzzling? What have they tried?
- Suggest a ridiculously wrong approach and have them explain why your suggestion does not make sense. Then ask them to try something that seems more reasonable.

Some groups finish quickly. Other groups are working at a very slow pace.

Suggestions:

Handling slow groups:

- Determine the reason a group is working slowly. Are they productively engaged? If not, see above for tips on engaging students who are sitting and staring.
- Determine if it is really necessary for all groups to finish all problems before closure. For groups that are working slowly, pick a few key problems for them to do about 10 minutes before you plan to pull the class back together for closure. Make sure that you involve these students in the class discussion even if they didn't finish the activity.

Handling fast groups:

- If a group has finished, assign them one of the problems to present to the class. They can make a poster or prepare clear notes for the document camera.
- Use students in a group that finishes quickly as roving tutors. If you do this, don't let them take their completed work with them when they provide help; this will prevent copying. Do a dry run with them to emphasize that they should not be just giving answers. The goal is for everyone to understand the concepts. If students are reluctant to tutor, remind them that those who teach learn the most. This is an opportunity to deepen their own learning.
- Reshuffle groups that have finished quickly, so that they can compare answers and approaches. (Let slower groups continue working.) Challenge them to come up with multiple approaches or explanations to key problems.

Summary of Interactive Strategies

Throughout the facilitation notes, we have used a variety of low stake collaborative strategies, in addition to group work. Here we highlight the purpose and logistics of the most frequently used strategies.

Think-Pair-Share

What is it?

Think-Pair-Share is a quick strategy that engages every student. It helps students bolster their confidence in their thinking and prepares them for participating in class discussion.

How it works:

Instructor poses a question. Students have quiet individual think time (1-2 minutes), then pair up with a neighbor to discuss their ideas (1-2 minutes), then instructor leads a class discussion in which students are randomly selected to share their thinking with the class.

Speed-dating

What is it?

Students pair off to discuss a problem, then rotate to a new partner to discuss the same problem or a new problem.

Speed-dating engages every student. It is a low stakes way for each student to improve his/her explanation or solution through repeated quick practice rounds. When students know that they will have to speed-date after group work, it can also improve engagement and individual accountability during group work.

How it works:

- Give clear instructions to students at the start because the room will get very loud once speed-dating begins. You will need a method that signals "switch partners," such as flicking the lights or ringing a loud bell. You should also describe what students will discuss in each round of speed-dating before speed-dating begins.
- For the initial pairing, tell students to form two lines down the middle of the classroom and pair up with a person opposite them in the other line. Make sure everyone has a partner. If there are an odd number of students, you can join the speed-dating.
- Tell students that they will have 3-minutes for their "date" to discuss a problem, then you will signal that it is time to switch partners.When the light flicks (or bell sounds), they begin another "date" and discuss the next problem (or the same problem again.)

- Explain how the switching will work: one line will remain stationary and the other line will move two spaces toward the front of the room, so the two people at the front of the moving line will walk all the way to the other end. (Why two spaces? This helps break up cliques so that students are forced to work with people that they may not know.)

Ambassador Exchange

What is it?

With an Ambassador Exchange, each group exchanges ideas with two other groups. We primarily use Ambassador Exchange after group work to allow groups to checks their work with other on a subset of problems.

- The visit: The Ambassador visits a host group and shares out. The host group also shares with the Ambassador.
- The return: The Ambassador returns to his/her group and shares what (s)he learned. The group also shares what they learned from their visiting ambassador. In this way each group has received input from two other groups.
- After the visit and return, check with the class to determine if there are lingering questions.

How it works:

- Number the groups.
- Randomly select an ambassador in each group or choose a student that needs the learning opportunity afforded by being an Ambassador or let each group select their own ambassador.
- Determine the rotation. You can be intentional and have strong groups visit weaker groups or just send Ambassador from group n to group n+1 Modulo n.
- Write a map on the board so the Ambassadors know where to go, as shown below.
- Ambassadors visit host group and exchange information. (This can vary, usually 4-8 minutes to compare answers.)
- After visiting the host group, the ambassador returns to his/her home group. The ambassador and the group discuss what they learned (This can vary usually 4-8 minutes.) Do they want to change any of their answers?
- Discuss lingering questions as a class. (This is often not needed after an Ambassador Exchange.)

Whole Class Quiz

What is it?

A Whole Class Quiz is a low stakes classroom assessment technique that can help increase individual accountability and engagement. It is also a good way to check for understanding and surface common misconceptions. Keep it light and fun.

How it works:

The class starts with a score of 10.

Pose the first question. Give students a minute to think about the answer, then randomly call on a student to answer. If an answer is wrong, call on another student at random to correct the answer. For each wrong answer that is not corrected, deduct 0.5 points and give the right answer. If the class score dips below an 8.0 or so, add opportunities for bonus points.

Either record the score in the Canvas gradebook or give out a rewards, e.g. candy or a bonus point on the next Group Quiz.

You can also do a Whole Class Quiz in pairs.

Module 1: Cereal Poster Activity

On the first day of class students work in groups on Module 1. This is an opportunity to begin to establish class norms and expectations around group work.

In order to complete the Cereal Poster Activity, you will not have time to go over your syllabus. Distribute your syllabus and assign it for homework. In the second class session, there is time for discussion of the syllabus.

What materials are needed?

- Module 1 (pages 3-5 of Math 100/110S packet) – one copy per student in case students have not purchased the activity packet
- Poster paper (one sheet for each group of 4 students)
- Graph paper and blank notebook paper (several sheets of each for each group of 4 students)
- Tape
- Markers (2-3 for each group of 4 students)
- Post-its (3x3in, two colors; for a class of 32, 168 post-its in each color)

Module 1 has two parts. We have allotted about 95 minutes for this activity.

Overview of timeline for entire activity

- Part 1 (20 minutes): Groupwork 10 minutes, discussion 10 minutes
- Part 2 (75 minutes): Set-up (5 minutes), groupwork and poster construction (30 minutes), gallery walk (20 minutes), discussion (20 minutes)

Part 1 (about 20 minutes)

Group work (10 minutes)

This is the first group work students will do and it does not involve any math or statistics. Hopefully, this will decrease students' fear or apprehension about the class and about working together in groups.

- Randomly assign students into groups of 4. See the FAQs in the Canvas Instructors Resources for instructions on how to use Canvas to randomly assign groups.
- Students should introduce themselves to their group mates and discuss questions 1-4.

Discussion (10 minutes)

Here are some suggestions for facilitating the 10-minute class discussion:

For each question, randomly choose a group to begin the discussion. After each response, follow up to engage other students and to reinforce class norms for discussions. This should go quickly (about 2.5 minutes per question.)

For example,

- Call on other students to paraphrase what another student has said. This communicates to students that need be listening to each other.
- Call on students to add to the explanation given or to provide an alternative perspective. Do this for both good explanations and for explanations that need improvement. This encourages students to respectfully agree or disagree with a peer's idea.
- Give encouraging feedback to everyone who talks, e.g. "you did a nice job explaining the multiple perspectives in your group," or "I like how you gave a specific example to illustrate your point," etc.

Part 2 Analyzing the cereal data (about 100 minutes)

<u>Timeline for Part 2:</u>

- Set-up (2 minutes)
- Group work and poster construction (40 minutes)
- Set-up for gallery walk (3 minutes)
- Gallery walk (25 minutes)
- Discussion (25 minutes)

The learning goals here are mainly affective. Students will quickly get the message that there is no procedure to remember or to mimic; they must use their own reasoning to make sense of the data, and this is the orientation we want them to have throughout the semester.

Other learning goals that reinforce class norms:

- Move away from "show me" and toward "what makes sense to me;"
- Practice thinking and communicating in a low-stakes environment;
- Take risks;
- Practice listening and responding;
- Gain respect for and learn from peers.

<u>Set-up for group work</u> (about 2 minutes)

Call on a few students to read the research question and instructions for Part 2 out loud; this will focus the class. Emphasize that this is a preliminary investigation, so we do not expect a complete or thorough analysis of the data. Let them know that they will have only 30 minutes to formulate a draft answer and make a poster.

<u>Group work and poster construction</u> **(**about 40 minutes)

- *Tip:* Allot about 10-15 minutes to make the poster. You may need to stop the small group discussions prematurely to get them to make their poster.
- *Tip:* To keep everyone in the group active during the making of the poster, hand out blank notebook or graph paper. Some group members can make a graph on the graph paper, or write up the conclusion on the notebook paper, and tape it to the poster; other group members can finish calculations.

<u>Gallery walk</u> (25 minutes)

- Dry run (5 minutes): Group's presenter practices and gets feedback from the group prior to the gallery walk.
- Gallery walk (20 minutes): Five or six rounds @3 minutes each.

Instructions to students:

  - One student will be the presenter. (The instructor can choose the presenter using random assignment or choose someone who looks disengaged or lost.).
  - Groups have 5 minutes to do a dry run. This gives the presenter time to practice and get feedback from the group.
  - Groups will rotate together (minus the presenter) when the lights flicker (or alarm sounds). Each round will last about 3 minutes.
  - Before rotations begin, give stacks of the two colors of post-its to each presenter.
  - Each student listening to a presentation will provide feedback to each presenter on post-its. One color is for a kudo: a positive note that captures something that was clear or compelling in the analysis. Another color is for a piece of constructive feedback: a note that highlights something that was unclear or perhaps incorrect or something that could be expanded in the analysis. Each student must leave two post-its (one of each type) at each poster.

<u>Important notes</u> for the gallery walk:

1) Students may be disappointed that they cannot see all of the posters and presenters may feel frustrated that they are not seeing any posters.

Quickly discuss the advantages of each role:

- Each presenter will do his/her presentation numerous times. This allows the presenters the opportunity to improve their understanding of their group's analysis and to improve their analysis and explanation using feedback.

- Rotaters get to see alternative perspectives and bring those ideas back to the group. They also play the crucial role of providing feedback to other groups.

2) Monitor the posting of feedback during the first two rounds. Visually check to see if there is an equal number of post-its in the two colors at each poster and that the number of post-its increases by 6 each round (3 of each color, left by each student.)

   If necessary, stop the gallery walk and review the instructions for feedback. Insist that each student leaves both a kudo and a piece of constructive feedback at each poster.

   This is really important because it helps students learn to mentally engage with a presentation, to treat their peers' ideas with respect, and to learn to disagree in positive and constructive ways. Peer feedback is an important part of the course. Students will need to be able to give constructive feedback during group work in class and in Canvas discussions.

<u>Closure for the entire Module 1 activity</u> (about 25 minutes)

- Ask groups to return to their own poster.

- Review feedback (5 minutes): They will naturally begin to look at the feedback on the post-its. They will usually also begin to explain what was on other posters to the presenter, who will probably be asking about this.

- Discussion (about 20 minutes): Students remain at their posters for whole class discussion.

    - Ask: What is the answer to the question? (e.g. Are child cereals less healthy than adult cereals? Raise your hand if you think the answer is Yes, No, Uncertain.)
    - Ask, "What evidence do we have in support of this answer?" Call on students to summarize evidence that they found compelling. Let them point to posters for reference.
    - After allowing time for a summary of evidence for each position, switch the conversation to feedback. Ask each presenter to share the <u>most helpful</u> piece of feedback that they received. The more specific the feedback, the more helpful it is. Emphasize that feedback is an invaluable way to improve your work and that it is a fundamental part of working productively in teams.

Setting Group Norms

Do this activity prior to the Module 2 activity. This activity is not in the students' packet.

Learning goals:

- Students can explain how group work promotes deeper learning and promotes powerful skills for life and work.
- Students develop norms for working together in groups.

Estimated time (25 minutes)

Overview for the instructor

This course relies heavily on collaborative group work. Many class assignments involve group work: class activities, Group Projects, Group Quizzes and the final project.

Group work has many benefits in producing and deepening learning, but it also requires students to work together in productive ways.

Many students do not like group work, so it is important to help them understand why we are using group work and to allow them to establish their own ground rules for productive engagement with their peers.

Rationale for group work (5 minutes)

Spend about 5 minutes discussing the rationale for using group work. You can reference the tables and diagrams in the Module 2 activity, but don't have students work on that activity yet.

Here are some ideas for establishing a compelling rationale for group work:

<u>Build important skills for life and work</u>: Project the Fortune 500 skills (Module 2 activity #1) and note that over time employers at the most profitable companies have come to value the ability to work in teams, problem-solve, and communicate verbally. There is no course in college that teaches these skills alone, but you will develop these skills through group work in this course.

<u>Retain more of what you learn</u>: Discuss the Cone of Learning (Module 2 activity #2). With a lecture you retain only 50% of what you hear and see. But with group work, you will be talking and doing, which leads to greater retention.

<u>Tell a personal story that reinforces the importance of group work:</u>
Some students feel that group work is "like high school," so share a story about how group work is integral to the workplace or to your experiences in graduate school.

Example: I hate to work in groups. I usually feel that I can accomplish more, and more quickly, if I work alone. However, my job involves committee work. I do not get to pick who is in my department or who is on a particular committee, but I have to work productively with these people to make decisions that affect the workplace and students.

Another example: Some people think that group work is only done in K-12, but in graduate school at UCB I worked in groups all the time. I am not sure that I would have survived it if I had not found other people to discuss ideas with.

Develop group norms (20 minutes)

Randomly assign students to groups of 4.

Why we hate group work (10 minutes)

- Group work (4 minutes): Students make a list of all the things that they hate about working in groups. After about 3 minutes, tell groups to pick their top three reasons for hating group work.

- Whip around (6 minutes): Call on each group to report one thing they hate about working in groups. No repeats allowed. Make a list on the board using their language.

- Allow students to add anything that is missing that they feel strongly about.

How we can be part of the solution, not part of the problem (10 minutes)

- Group work (4 minutes): Students make a list of things that people can do to address the problems generated previously. After about 3 minutes, tell groups to pick their top three things.

- Whip around (6 minutes): Call on each group to report one thing. No repeats allowed. Make a list on the board using their language.

- Allow students to add anything that is missing that they feel strongly about.

- You can also add to the list. For example, many times students will not include a norm about cell phone use. You can add “be present during class, not on the phone” or something like that.

For the next class period, type up the norms and display them on the document camera during group work throughout the semester.

If students are violating a norm, just remind them that these are the things the class developed to make group work more productive. Productive groups = deeper learning.

Later in the semester, if a problem arises with group work, e.g. some students are disengaged. You can revisit this discussion. Note the problem you are observing and ask the class if additional norms need to be established to help address the problem.

Sample class norms:

Step up, step back.
Be respectful. Be kind.
Be patient.
Listen to others' ideas.
Disagree with ideas not people.
Help each other learn; helping is not the same as giving answers.
No one is done until everyone is done.
Stay focused.
Celebrate mistakes as an important part of learning.

Syllabus quiz

Learning goal: Understand class policies.

Estimated time: 15 minutes

Instructors often feel the need to go over the syllabus because they view it as “a contract” and they want students to understand the class policies. However, going over the syllabus can be boring and it can also be too much information too early. As the semester progresses, students will often forget about the syllabus and aggravate instructors by not remembering class policies around late work, attendance, exams, etc.

Instead of going over the syllabus, we recommend assigning the syllabus as homework on Day 1. On Day 2, do a fun Whole Class Quiz on key information. Then, throughout the semester, return to review information in the syllabus as it is needed. For example, review late work policies when you make the first assignment. Review how grades are determined after you enter the first grades. Review exam make-up policies when you are preparing the class for the first exam, etc.

<u>Whole Class Quiz in groups</u>

A Whole Class Quiz is a low stakes classroom assessment technique that can help increase individual accountability and engagement. It is also a good way to check for understanding and surface common misconceptions. Try to keep it light and fun.

Randomly assign groups of 4. Give everyone a minute to introduce themselves.

The class starts with a score of 10. Students can look at the syllabus during the “quiz.”

Pose the first question. Give groups a minute to discuss the answer, then randomly call on a group to answer. If an answer is wrong, call on another group at random to correct. For each wrong answer that is not corrected, deduct 0.5 points and give the right answer. If the class score dips below an 8.0 or so, add opportunities for bonus points.

Either record the score in the Canvas gradebook or give out a rewards, e.g. candy or a bonus point on the next Group Quiz.

Some suggested questions:
- In-class work accounts for what percentage of your grade?
- Canvas homework accounts for what percentage of your grade?
- What assignments can be redone to improve your score (if you finish by the due date)?
- This course includes 2 hours a week of math lab. What will you be doing in the math lab?
- Where can you get help?

4.1 Distribution of Quantitative Data (Introduction)

Learning Objectives:

- Develop a way to describe and distinguish graphs of a quantitative variable
- Identify reasonable explanations for what might explain the differences seen in different data sets.

Estimated time: about 75 minutes

Set up (about 7 mins)

In this activity students will compare dotplots. This is their first introduction to dotplots and to the concepts of shape, center and spread of a distribution.

We do NOT recommend that you show them how to make a dotplot or that you go over the definitions before beginning of the activity. Why? Doing so will only reinforce common student expectations that you will demonstrate and they will mimic, that you will give definitions and they will memorize. Instead we want students to grapple with seeing distributions as a statistician does and to think about what might cause variability in data.

Instead, keep the set-up short. You can simply show this short movie and use class time to check for understanding after students have worked on the activity.
https://screencast-o-matic.com/watch/cqV20x3IAi

This movie will connect the thinking that students tend to do in the Cereal Posters (comparing two cereals or calculating means) with the type of thinking that statisticians tend to do. This will set them up for richer discussions as they work through this activity.

Implementation (about 30 minutes)

- Randomly assign groups of 3 to 4 students.

- Start by calling on different students to read the Introduction to out loud. This will focus the class. (3 mins)

- Group work on problems 1-3

Possible areas of difficulty for Problems 1-3:
2c) Two of the three possible explanations are clearly not right (the 1st two), but the third is not intuitively obvious. If students argue that none of the options make sense, ask them to provide another possible explanation for the differences in the graphs.

3b) It might be hard to come up with a situation that accounts for the differences in the graphs. Encourage students to make up scenarios and discuss what the distributions might look like for different scenarios. For example, what would the distribution look like for a really easy exam? For a really hard exam? Then students can fine-tune their descriptions. For example, Set G could be scores from an exam with one really hard question worth 20 points that no one got, but the other questions fairly easy (hence the skew to the left.) Set H might be graded on a curve. Set I could be scores from an exam where 70% of the questions were pretty easy but the last 30% were harder.

3c) Students may ask about vocabulary describing common shapes. Refer them to the appendix.

For groups that finish quickly, ask them to try to use statistical vocabulary, from the table given at the beginning of the activity, in their explanations.

Closure: Discussion of problems 1-3 and mini-lecture (about 30 minutes)

Here are some ideas for closure. Choose between an Ambassador Exchange or a Class Discussion (20 minutes.) Leave about 5 minutes at the end to review vocabulary:

1) <u>Ambassador Exchange</u> (20 minutes total)

With an Ambassador Exchange, groups check each other's work.

Overview of the Ambassador Exchange:

- The visit: The Ambassador visits a host group and shares out. The host group also shares with the Ambassador.
- The return: The Ambassador returns to his/her group and shares what (s)he learned. The group also shares what they learned from their visiting ambassador. In this way each group has received input from two other groups.
- After the visit and return, check with the class to determine if there are lingering questions.

Logistics for an Ambassador Exchange:

- ◊ Number groups.
- ◊ Randomly select an ambassador in each group or choose someone who needs the learning opportunity or let each group select their own ambassador.
- ◊ Write a map on the board for the ambassador exchange to tell Ambassadors where to go.
- ◊ Ambassadors visit host group. (7 minutes to compare answers.)
- ◊ After visiting the host group, the ambassador returns to his/her home group. The ambassador and the group discuss what they learned (7 minutes.) Do they want to change any of their answers?
- ◊ Discuss lingering questions as a class (6 minutes)

2) Discussion (20 minutes total)

- Ask each group to identify one item they would like the class to discuss. Repeats OK. Discuss most popular items first, in case you run out of time. Spend only 15 minutes here. You can answer questions and check their work when you circulate as groups work on the next set of problems.

- Why not work through all of the problems on the board? Because students will get the message that in the future they do not need to do the work; they will just wait for you to do it for them.

- Randomly choose a group to begin the discussion. They can provide an answer or share a part of their group discussion of the problem if they did not come to agreement on an answer. After each response, follow up to engage other students and to reinforce class norms for discussions.

  For example,
  - Call on other students to paraphrase the answer and/or rationale. This communicates to students that need be listening to each other.
  - Call on students to add to the explanation given or to provide an alternative perspective. This encourages students to respectfully agree or disagree with a peer's idea.
  - Praise students for perseverance, for wrong answers backed up by sound reasoning, for correct answers that reflect the way a statistician would think

End the lesson with a mini-lecture to introduce vocabulary (10 minutes) – There is a vocabulary list in the appendix of the packet.

Talk through the problems using the vocabulary of shape, center and spread. Here are the main points:

Problem 1: These distributions have the same shape, and the same amount of spread (overall range: difference between lowest and highest score). But the center shifts. The center is a representative score that can be used to summarize the entire classes performance on the exam.

Problem 2: These distributions have the same shape (symmetric with a central peak) and the same center (average is used to summarize the data). But the spread differs. For example, there is a much bigger gap between the lowest performing student and the highest performing student in Set F.

Problem 3: These distributions differ in shape. You can reinforce some vocabulary here: skew left, skew right, symmetric. But they have the same average score (same center) and the same spread (range is 30).

4.2 Dot plots

Specific Learning Objectives:

- Distinguish between categorical and quantitative variables;
- Identify graphs that represent the distribution of a quantitative variable;
- Analyze the distribution of a quantitative variable using a dot plot. Describe the shape, give a general estimate of center, and determine the overall range.

Estimated time: about 90 minutes

Suggested set-up (15 mins)

Instead of lecturing on the definitions of categorical and quantitative variables, get students thinking.

- Open a StatCrunch spreadsheet of Cereal.txt. Ask students to identify variables that could be used to make a dotplot. Call on students to explain how they knew. Expand on their answers by saying that a statistician calls these variables "quantitative variables". Quantitative variables have numerical values that can be averaged.

- Show how a categorical variable can be used to separate the data into groups to create two or more dotplots. (From the dotplot window, open Options, Edit. In Group by, choose a categorical variable, such as Shelf.) Discuss the meaning of "categorical".

- Imagine the LMC application. Ask students to think of a variable that has numerical values that cannot be averaged (zip code, phone numbers, student ID numbers). Point out that these are not quantitative variables. Some can be considered categorical, such as zipcode which categories us by geographic location.

Remind students that statisticians will describe quantitative data using SHAPE, CENTER and SPREAD. Allow students to relate these words to everyday language but don't worry if they are a bit off track. (ex. "When do you use the word *spread* in everyday language and what does it mean in common English?" and then "What does it mean for a statistician?")

Suggested implementation (about 60 mins)

Randomly assign groups of 4. Call on a student to read the Overview out loud to focus the class.

Possible areas of difficulty:

1a) Are the individuals in this data set individual cars or car make/models? Students could make good arguments for either position, and this is fine. If the individuals are car make/models, then there are 4 variables. If the individuals are individual cars, then there are 5 variables.

1c) Engine is a categorical variable despite the use of numbers.

2-3) Some questions do not have a “right” answer. As you circulate and check in with groups, focus on getting students to use the data to justify their answers.

5-6) Check for accuracy. Many students will think left-skew means most of the data is on the left. You can think of skew as deviation from symmetry. If data is symmetric and you pull some of the data out to make it skewed, the direction that you pulled is the direction of the skew.

7c) The interval of typical mpg for these cars is a judgment call, but should not be min to max. Again it is important for students to be able to justify their answer based on the data; answers will vary.

Closure (about 15 mins)

We recommend more in depth work on #8 during closure. Canvas provides many practice opportunities for the concepts covered in this activity, including problems like #8, but #8 is involves the most complex set of skills.

- (3 mins) Project this example of how an observation about shape can be used to support the position that child cereals tend to have more sugar:

  Child cereals tend to have more sugar. For child cereals, the shape of the distribution is skewed to the left, which means that only a few child cereals have small amounts of sugar. This is the opposite for adult cereals where the shape is skewed to the right, which means that only a few adult cereals have large amounts of sugar.

  Highlight that this paragraph is good because it is more than a list of observations. Specifically, the following things make it good: (1) the correct use of statistical vocabulary, (2) the comparison of adult and child cereals and (3) the description explains how the shape supports the thesis that child cereals have more sugar.

- (5 mins) Give students time to revise their work on #8 into a paragraph that addresses this question: *Which tends to have more sugar, adult or child cereals?* They should use shape, center and spread in their paragraph(s).

- (2 mins) Groups pick out the best paragraph from their group.

- (5 mins) Students share paragraphs using the document camera.

High Quality Feedback Activity

Learning goal: Write high quality feedback for discussions in Canvas and for poster sessions.

Estimated time: 15 minutes

Set-up (5 minutes)

- Project Module 4 Dotplots (2b of 2 discussion 1) in Canvas.
- Click on the link to general information about Learn by Doing discussion board exercises.
- Review the rubric and highlight the role of high-quality feedback. Six of the 10 points are tied to feedback.
- Review the section called High Quality Feedback.

Group work (5 minutes)

Project the following initial post and the feedback. Groups discuss how to improve the feedback.

Initial post:
Child: Shape is stacked. Center is 110. Spread is 90 to 120.
Adult: Shape is spread out. Center is 100. Spread is 50 to 160.

Feedback:
Write more. Your answer is too short.

Discussion (5 minutes)

Call on groups to read their improved feedback or to project it with the document camera.

Name the qualities of good feedback as you hear it. For example:

- Names what is done well
- Gives specific suggestions for improvement
- Is supportive and encouraging in tone
- Encourages the use of course vocabulary
- Highlights the parts of the prompt not addressed by the post

4.3 Histograms

Learning Objectives:

- Distinguish between categorical and quantitative variables;
- Identify graphs that represent the distribution of quantitative variables;
- Analyze the distribution of a quantitative variable using a histogram;
  Describe shape, give a general estimate of center and overall range, and calculate relevant percentages;

Estimated total time: 100 minutes

This activity introduces students to histograms. You will not have time to complete this entire activity in class, so we recommend the short cut described in these notes.

Set Up: (about 25 minutes)

We do NOT recommend that you demonstrate how to make a histogram by hand because students are not required to construct histograms in this activity (or in the course). Why? Because histograms are not useful for summarizing small data sets. Eventually, students will use StatCrunch to make histograms for large data sets. In addition, we have seen students get so bogged down in the nuances of histogram construction (e.g. left- and right- endpoints etc.) that they don't learn to analyze and interpret histograms well.

(7 mins) Show this short movie to demonstrate the relationship between dotplots and histograms and to motivate the need for a histogram. (6:17 minutes)

http://somup.com/cqV00xnDpL

(18 mins) After the movie, interactively work through the 1st problem with the class.
Note: We use the convention of including the left-hand endpoint in the bin, e.g., X=50 is included in the 2nd bin.

Start with a few quick comprehension questions:

- How many students scored an 80 or between 80 and 90? [Answer: 2]
- What were their scores? [Answer: We don't know.]
- Did anyone score a 35? [No] A 75? [We don't know.]

Use Think (3 mins) –Pair (3 mins)–Share (9 mins) to work through the Overview problem.

Implementation (about 45 minutes)

Divide students into random groups of 4 to complete the activity.

As you circulate,

- prod for explanations
- check for accuracy and highlight areas that you want groups to revisit

Areas of potential difficulty:

#2: Students may get stuck here. It may help for students to sketch a dot plot for each description with numbers on the horizontal axis, then match their dot plots to reasonable histograms. There may be some debate on these, too. Focus on strong reasoning in support of answers, rather than one right answer. For example, dates on pennies in my car ashtray could be skewed right if I assume that old pennies are rare. But there have been efforts to decrease the production of pennies in the U.S. because it costs more to make a penny then it is worth, so the distribution could be skewed left if I assume that newer pennies are rare.

#3b: If students do not identify a reasonable interval of typical scores, try suggesting a bad answer, e.g. 64 to 80, and ask why scores in that interval are not typical, then ask them to make a better recommendation. Reassure them that we are just estimating here. There can be multiple reasonable answers.

#4e: Students may ignore the definition of the base group (5^{th} graders who are not following ADA recommendations). They may just calculate the percentage of 5^{th} graders carrying more than 25% of their body weight. If they do this, point out that they answered the question, "What percentage of 5^{th} graders carrying more than 25% of their body weight?" Ask how the question in (4e) differs from the question that they answered.

#5a&b: There are many ways to use percentages to support an answer here. Work with students on compelling use of the data to support an answer. For example, for (5a) comparing the percentage with A's is not that compelling: Front 3/51 = 0.59, Back 2/46 = 0.04. But comparing the percentage with A or B is more compelling: Front 41/51 = 0.80, Back 24/46 = 0.52.

#5c: We have found that students think it is "unfair" to compare groups of different size, even when we use percentages. Keep an eye out for this misconception. Explain how percentages can be interpreted as scaling the groups to 100 individuals for easy "fair" comparison.

For groups that finish early, do one or more of the following:

- If a group has finished, assign them one of the problems to present to the class (maybe #8). They can make a poster or prepare clear notes for the document camera.
- Use students in a group that finishes quickly as roving tutors. If you do this, don't let them take their completed work with them when they provide help; this will prevent copying. Do a dry run with them to emphasize that they should not be just giving answers. The goal is for everyone to understand the concepts. If students are reluctant to tutor, remind them that those who teach learn the most. This is an opportunity to deepen their own learning.

- Reshuffle groups that have finished quickly, so that they can compare answers and approaches. (Let slower groups continue working.) Challenge them to come up with multiple ways to support their thesis in #8.

Suggested closure (30 minutes)

While you were circulating, you have already checked for accuracy and understanding, so there is no need to go over all of the answers now. In addition, you want to avoid going over answers with the class because this communicates to students that they don't need to do the work; they can just wait for you to go over the answers.

Here are several ideas for closure. Choose one or do several if there is time.

<u>Discussion</u>:

Discuss a few items as time permits. Ask each group to identify one part that they would like to have the class discuss. Alternatively, discuss the item that you observed was the most problematic for students.

<u>Ambassador exchange</u>

If a group has struggled, intentionally design the exchange so that they get feedback from stronger groups. See Summary of Interactive Strategies for details on how to run an Ambassador Exchange.

If you are pressed for time, have groups identify a few problems on which they want feedback. The Ambassador can seek out information on those problems when (s)he visits the host group, and the home group can ask their visiting Ambassador about those problems, too.

<u>Speed-dating</u>

If you have observed individual students who are not participating in their groups, try Speed-dating to increase individual accountability and to give quiet students a low stakes opportunity to discuss their understanding. Speed-dating also gets everyone up and moving around, which is a nice way to wake everyone up.

See Summary of Interactive Strategies for details on how to run Speed-dating.

If you are pressed for time, pick a few problems for students to discuss based on your observations of what students struggled with during group work. Use one problem per speed-dating round. You can use the same problem for multiple rounds, too. Alternatively, allow students to pick a problem to discuss when they are in the "date."

4.4 Module 4 Lab

Learning Goal: For the distribution of a quantitative variable, describe the overall pattern (shape, center, and spread) and striking deviations from the pattern.

Specific Learning Objectives: Compare and contrast the distributions of a quantitative variable for two groups using histograms. Describe shape, give a general estimate of center and the overall range, and calculate relevant percentages.

Estimated time: 60 minutes

If you have been doing a lot of group work, you might want to facilitate this lab differently.

Here are some ideas:

<u>Individual work, Speed-dating, individual revision</u> (about 40 mins)

- Individual work on #1, #2a-d (15 mins)

- Four rounds of speed-dating focused on #1 and #2a-d (about 15 mins)
  Do multiple 3-minute rounds on the same problem so that students gain confidence in their answers and explanations:
    - Round 1: Compare #1 a-c.
    - Round 2: Compare #1 a-c again.
    - Round 3: Compare #2 a-d.
    - Round 4: Compare #2 a-d again.

- Individual work (5-10 mins): students revise answers based on what they learned in Speed-dating. Students who do not need to revise serve as a Roving Tutor.

<u>Think-Pair-Square on #2e</u> (20 mins)

- Individual work on #2e (5 mins)
- Pair off to discuss #2e (5 mins)
- Square off (two pairs join) to discuss #2e again (5 mins)
- Individual revision time (5 mins): Students who do not need to revise serve as Roving Tutor.

Collect and grade. Papers should be good and thus easy to grade at this point!
OR Record an effort grade and distribute solutions.

How Learning Happens

Learning goal: Understand how the various pieces of the course fit together to foster statistical thinking, deeper learning and the ability to apply what we learn to real life.

We suggest that you do this short activity after students have completed Mods 1-3 in Canvas, Setting Group Norms and 4.1.

Estimated time: 15 minutes

Overview for instructors:

Students will be expecting a traditional math classroom: lecture, a set of examples, homework problems that look just like the examples. It is important to talk with them about how learning happens in this student-centered classroom so that they understand why we are learning through group activities followed by Canvas work and why you are not delivering long lectures.

They also need to be reassured that deeper learning can happen in this format, and that you will be there to monitor and guide. When students understand the rationale and how all the pieces fit together, they are more likely to productively engage.

If students have completed Module 2 in Canvas, then they read about the following ideas that you can reinforce here:

- This course requires thinking about ideas, instead of memorizing procedures.
- Growing the brain through struggle and challenge.

Materials:
This activity will go quickly if you prepare a document that you can project that has three headings and the list of learning goals. When groups share out, you can simply cut and paste the learning goal below the appropriate heading.

Set-up (3 minutes)

Randomly assign groups of 4.

Give students a short overview of the purpose of this activity. Something like the following:

- We are going to spend a few minutes talking about how learning happens in this class.

- High school algebra is about equations and procedures. College statistics is about analyzing data to make decisions. This involves mathematical models, but the focus is not calculations or procedures, and there is not always one right answer. Instead we focus on analyzing data, understanding variability, and making inferences.

- For this reason, the best way to learn statistics is to do it. Copying notes from the board, mimicking procedures from examples, drill and kill is not going to make you into a powerful statistical thinker. Instead, this class is set up so that you will talk the talk and walk the walk of a statistician.

- This class involves class activities, Canvas homework, Labs/Group Projects/Group quizzes. Let's think about the role of each of these in your learning.

Group work (3 minutes)

Project the three headings and the list of learning goals. Give groups a few minutes to match learning goals to headings

<u>Three headings:</u>

1) Class activities: Making sense of new ideas
2) Canvas work: Solidifying new ideas
3) Labs, Group Projects, Group Quizzes: Pulling it altogether

<u>Learning goals:</u>

- Prepare to understand the material in Canvas
- Introduce new concepts and grapple with new ideas together
- Read concise explanations of the ideas discussed in class
- Try things out, ask questions
- Apply what you learned to more challenging real-life contexts
- Discuss connections with previous ideas
- Practice solving "word" problems with your peers
- Practice and assess your learning with lots of low-stakes quizzes and some discussions
- Pull larger chunks of material together
- Prepare for exams

Report out (3 minutes)

Whip around. For each heading, call on groups at random to identify learning goals for that heading. Each group can give a new learning goal for that heading or move a learning goal to another category. Since students have not experienced the full learning cycle, there may be some disagreement, but the wording of the learning goals is suggestive.

After all of the learning goals are assigned, reshuffle if necessary to match the lists below:

Class activities: Making sense of new ideas

- Introduce new concepts and grapple with new ideas together
- Try things out, ask questions
- Discuss connections with previous ideas
- Practice solving “word” problems with your peers
- Prepare to understand the material in Canvas

Canvas: Solidifying new ideas

- Read concise explanations of the ideas discussed in class
- Practice and assess your learning with lots of low-stakes quizzes and some discussions

Labs, Group Projects, Group Quizzes: Pulling it altogether

- Apply what you learned to real life situations
- Pull larger chunks of material together
- Prepare for exams

Closure (about 6 mins)

Think (1 min)–Pair (1 min)–Share (2 min): In class, why do we require you to grapple with new ideas in groups instead of just listening to a lecture and taking notes?

Add these ideas if they do not come up in the share out:

- Grappling grows your brain! It is essential to growing the neural connections that produce deeper learning. At the gym, you push yourself and sweat to get into better shape. You don’t just watch your trainer work out. No pain, no gain, as they say. Group work is our work out.
- Group work requires us to “do and say.” In the Cone of Learning “Do and Say” makes you retain more of what you learned.
- Group work makes learning visible so that I, as your instructor, can monitor and guide your understanding by asking questions that will help you move forward.
- Group work builds skills for life and work: Ability to Work in Teams, Ability to Problem Solve, and Ability to Communicate Verbally (from the Fortune 500 CEO list.)

(2 mins) Close by reassuring students that after group work, there will be an opportunity for you to assess your own learning. This may involve comparing answers with other groups (Ambassador Exchange), working a new problem by yourself and then comparing with another student (Think-Pair-Share or speed-dating), presenting a problem on the document camera, etc.

5.1 Measures of Center (Develop a Feel for Mean and Median)

Specific Learning Objectives:

- Find the mean and median from different representations of data.
- Develop number sense with mean and median by creating different data sets with a given mean or median.

Estimated total time: 90 minutes

We are now making the idea of center more precise by using the mean or the median. There is no formal work or introduction about mean and median prior to 5.1, but students have been estimating "center". It is important to repeatedly highlight that both the mean and the median are ways to describe a distribution using a single number, i.e. ways to identify a representative value.

Suggested set-up: (20 mins)

Use #1-3 as an introduction to the activity.

<u>Discussion of (1)</u> (about 10 mins)

Use QUICK Think-Pair-Share on (1a). Students may think that the longer the list of numbers, the larger the mean, so it is important to give them a few seconds to pick the Class with the larger mean BEFORE they have time to calculate, then discuss. To drive the point home, you could ask, which has the larger mean? {95, 95, 95} or {70, 70, 70, 70, 70, 70, 70}. Don't be surprised if some students need to calculate the mean to answer this.

For (1b) <u>quickly</u> calculate the mean AND the median for both Class A and B. Most students will already know how to do this. [Class A: mean 90, median 90; Class B: mean ~75.7, median 75]

<u>Discussion of (2) and (3)</u> (about 10 mins)

Use Think-Pair-Share. Students will often be baffled by these problems because no calculations are possible. This forces them to think about the mean as a representative value, a single number that summarizes the distribution.

Implementation: (40 mins)

- Randomly assign groups of 4.
- Students work on #4-9.
- Circulate to check for accuracy, particularly on #4-5. If groups are struggling with #4-5, intervene and ask questions to move them forward.
- As you circulate, look for groups that have successfully completed #6, #7 or #8. You will call on these groups for explanations during Closure.

Possible areas of difficulty:

#5: Students may be confused about what numbers to average. Encourage them to make a list of the data or make a dot plot.

#6-#8: These problems require students to play around. Students may ask for a procedure. Let them know that there is no procedure to apply. Encourage them to approach these problems by trial and error or by reasoning it through. Tell them that these problems are designed to build their intuition with regard to mean and median.

If groups finish quickly, assign them a problem, or part of a problem, to present from #6-8.

Closure: (30 mins)

Don't worry if some groups have not finished the activity. If everyone has gotten through at least (6a), you can start Closure. We suggest a combination of student presentations and a short assessment.

Student presentations of #6-8 (20 mins)

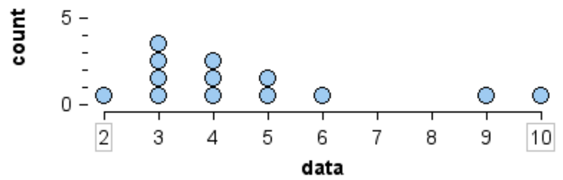
- If you are pressed for time, have students present on (6a) (6b) and (7) and skip (8).
- During the presentations, ask students to explain how they figured out a data set that worked.
- After the presentations, students should understand the following ideas. Emphasize these ideas as appropriate during the presentations:
    - A list of all 7's is the easy way to answer (6a); to find other data sets with a mean of 7 and a median of 7, make the same adjustments above and below 7 (e.g. if you change a 7 to a 5, change another 7 to a 9);
    - The median is the center of the list so it is unaffected by changing numbers below it or above it (keeping n=5), but the mean is affected by these changes.
    - To keep the mean constant, keep the sum constant when working with different sets of 5 number.

Skip #9, unless you have time to get some student conjectures on when the mean is not a good measure of center. We discuss the interplay of shape and center in Activity 5.3.

Assessment: (10 mins)

Use Think (1 min)–Pair (2 mins)–Share (about 7 mins).

Ann has the following data set. She thinks the mean is 61 and the median is 6. Without doing any calculations, do you think Ann's answers are reasonable? Or has she made a mistake? How do you know?



Big ideas for this discussion:

- The mean and the median are two ways to identify a representative value. Neither 61 or 6 looks representative.
- Clearly, 61 is too large to be the mean since all of the data falls between 2 and 10.
- The median is not the center of the number line or halfway between the min and the max. The median has to be less than 6 because most of the data lies below 6.

5.2 The Mean as a Balancing Point

Learning Goal: For the distribution of a quantitative variable, describe the overall pattern (shape, center and spread) and striking deviations from the pattern.

Learning Objective: Discover why the mean is called a balancing point.

Estimated time: 70 minutes

This activity is important because it gets students thinking about distances of data points from the mean, which foreshadows the development of standard deviation.

Suggested set up (about 5 mins)

Think (1 min)–Pair (1 min)–Share (3 mins) on this warm-up problem:

Without doing any calculations, determine which data set has the largest mean and the smallest mean.

A={70, 70, 70} B={70, 70, 70, 70, 70, 70, 70} C={70, 70, 0}

Suggested implementation (about 45 minutes)

- (20 mins) Randomly assign groups of 4 to work on #1-5. Groups working more quickly can work on #6-7.
- (12 mins) Ambassador Exchange to compare answers on #1-5.
- (5 mins) Clear up any lingering issues on #1-5.
- (8 mins) Groups work on #6-7. Groups that finish early can compare answers and prepare to present on the document camera.

Possible areas of difficulty:
1a) Some students have difficulty with generating a set of 8 scores that are the same with a mean of 60. Get them to try any set of 8 scores and make adjustments to raise or lower the mean, then call their attention to the requirement that all the scores have to be the same. This is usually enough to make the light bulbs go off.

3) Students may have difficulty adding signed numbers, even with their calculators. It may help to think of distances from the mean on a dot plot, with the sum of negative distances equaling the cumulative distance of data below the mean (hence the sum is negative.) Don't spend a lot of time on arithmetic rules for adding negative numbers. Focus on the big idea of summing distances below the mean and above the mean to show that the cumulative distances are the same and hence add to zero.

Suggested closure: (about 20 mins)

(10 mins) Student presentations on #6 and #7 using the document camera.

(5 mins) Show this movie to demonstrate geometrically how signed distances from the mean sign to zero

http://somup.com/cqVOclnbea (4:07 mins)

(5 mins) After the movie, Think (1 min)–Pair (1 min)–Share (3 mins) on #8 (the last question in this activity) to encourage class to ariculate what they have learned.

Module 2 Learning Strategies

Learning goal: Identify learning opportunities available in the course.

Estimated time: about 15 minutes

Do this activity after students have completed Modules 1-3 in Canvas. Students need some experience with Canvas exercises (quizzes and discussion) before they can reflect on learning strategies for the course.

<u>Group work</u> (10 minutes)

Randomly assign groups of 4 to work on #1-3.

<u>Discussion</u> (5 minutes)

Call on a group to answer #1. Ask other groups to contribute additional ideas. Add to their ideas if necessary. Repeat for the other two questions.

By the end of the discussion, we want students to see all of the opportunities for collaboration and teamwork (both in class – class activities, Group Projects, Group Quizzes, Final Project; and out of class – Canvas discussions and Labs).

Because Math 110 is essentially all "word problems," everything we do will require problem-solving and verbal communication.

There are many opportunities for receiving feedback and using it to make improvements:

- Class activities: feedback from group mates and instructor as you try out new ideas
- Group projects with poster presentations: feedback from class mates on your group's draft
- Canvas discussions: feedback from group mates to improve your work on discussion problems
- Canvas quizzes: immediate feedback and redo opportunities to improve your understanding and your score

5.3 Shape and Measures of Center

Learning Goal: For the distribution of a quantitative variable, describe the overall pattern (shape, center, and spread) and striking deviations from the pattern.

Specific Learning Objectives: Relate measures of center to the shape of the distribution. Choose the appropriate measure for different contexts.

Estimated time: **60 minutes**

Suggested set up (10 minutes)

In this activity we will examine how the shape of a distribution affects the mean and the median. It is important to reinforce the notion of "typical value" or "representative value" to help students understand why the mean may not summarize a skewed distribution very well.

Start with a short interactive review. Ask questions, such as

- How does a statistician describe a distribution? (Shape, center, spread, and outliers)
- What are some words we have learned to describe shape? (left- or right-skewed, symmetric with a central peak).
- What are two ways statisticians describe center? (mean and median)
- What are some things we have learned about the median? (Middle of the ordered list of data, physical center of the distribution, half of the data lies above and half below) About the mean? (How to calculate it, how to represent it using summation notation, balancing point—sum of the deviations above equals sum of deviations below, "fair share")

To transition to group work, say, "When is the median a better measure of center for a distribution? When is the mean better? Answering these questions is our focus today."

Suggested implementation (20 minutes)

Divide students into random groups of 4. Work on #1-4.
If groups finish early, they can work #5-7.

Circulate and check for accuracy in both answers and explanations. Work with groups as necessary to steer them in the right direction if they are off track.

Potential areas of difficulty:

#2d: Students may have trouble choosing whether the mean or median is a better measure of the strongly left skewed distribution. Sometimes students will argue that the mean takes all the values into account, so it is always better. In reply to this viewpoint, emphasize that we want to choose a value that summarizes the data well. Ask students to eyeball a representative value, like we did prior to learning about mean and median. They will probably pick 8. You can also say that we don't want one or two scores to have a huge affect on our choice of a "representative" value for the distribution. Why? This gives these two scores too much influence.

#4: Students may be troubled that they cannot calculate anything, but if they have answered #3 well, they should be able to use the shape to determine if the mean is larger than the median.

#5: If students are struggling with #5, ask them to sketch histograms or dot plots.
(5b) The wording of the M&M situation may be confusing. Students are confused as to whether they're weighing individual candies or the whole bag. We are weighing the 1-lb bag repeatedly. Sometimes the scale will record the weight slightly lower than 1-lb, other times slightly over. This is the nature of scales. There is a small bit of inaccuracy.

#6 ask how Bill Gates would affect the distribution of incomes.

#7 Keep an eye out for good answers here and call on these students during closure.

Suggested closure (30 minutes)

Choose one or more of the following, but leave 10 minutes at the end to discuss #7:

<u>Discussion of #4</u> (15 mins)

Discuss #4 as a way to summarize the point of this activity (about 10 minutes):

- In StatCrunch, pull up the Body Measurements data set. Make a dotplot of the ages.
- Ask students for their estimate of the median and their reasoning.
- Show the median on the dotplot (in the StatCrunch dotplot window, choose options, and check the boxes next to median)
- Ask why the median is not halfway between the minimum age of 17 and the maximum age of 67, which is 42. (This is because the median divides the distribution in half. Here the upper half of the data is more spread out.)
- Ask students whether mean is larger, smaller or about the same based on their answers to #4.
- Show the mean on the dotplot. Ask students to explain it makes sense that the mean is larger than the median. Let several students do this. (The older people affect the sum of the ages, so they pull the mean higher. However, if we lined everyone up by age, the older people would not affect the age of the person in the middle of the line.)
- Repeat for wrist girth, which is a roughly symmetric distribution. Emphasize that in symmetric distributions, the median and mean are equal, or close to equal.

OR

Speed-dating on #5 (10 mins)

Do several rounds of speed-dating using #5, then randomly call on students to answer #5.

OR

Snowball fight on #5 (10 mins)

A snowball fight is a low-stakes classroom assessment technique.

Logistics:

- Everyone writes their answer to #5 on a piece of paper.
- At your signal, everyone balls up their paper and throws it into the center of the room.
- Everyone retrieves a paper and opens it.
- Tally on the board: how many chose (a), (b) and/or (c). This is a save way to gauge if a more in-depth discussion is warranted.
- Elicit explanations. Students can share the answer on their snowball or share their own reasoning. This is a safe way to encourage students to participate in the class discussion.

(10 mins) End the lesson with Think (1 min)–Pair (1 min)–Share (5 mins) on #7

During the Share, call on many students at random to contribute.

In the discussion the big ideas to reinforce are:

- The “center” is a single value that we use to represent the entire distribution.
- Mean and median are tools for representing the center of a distribution.
- The mean may not represent a skewed distribution well because a few outliers can have a big impact on the mean. But outliers do not affect the median, so the median is a better choice to represent a skewed distribution.
- The mean is often pulled in the direction of the skew.
- If in doubt, plot the data to see which measure looks like a good representation of typical values.

6.1 Quantifying Variability Relative to the Median

Learning Goals:
- Create and interpret different graphs of a quantitative variable
- Summarize and describe the distribution of a quantitative variable in context. Describe the overall pattern (shape, center and spread) and striking deviations from the pattern.

Specific Learning Objectives:
- Use quartiles to quantify variability about the median.
- Create and interpret boxplots, relate boxplots to histograms and dot plots.

Estimated time: about 70 minutes

Suggested set up (about 15 minutes)

Do #1 interactively as a class. You could use quick iterative cycles of Think-Pair-Share to engage students.
- Think-Pair-Share on finding the median. Get someone to explain how to find it.
- Have other students read out loud the informational paragraph about finding quartiles.
- Think-Pair-Share on finding the quartiles. Get a few students to explain how they found the different components of the 5-number summary.
- Think-Pair-Share on understanding how to construct a boxplot.

Do not spend time on the nuances of finding the 5-number summary for an even number of data points. We will use StatCrunch to find 5-number summaries. Instead focus on the idea that quartiles divide the data into 4 groups of equal counts (same amount of data in each quartile). This is in contrast to histograms that divide the data into bins of equal width.

Suggested implementation (about 30 minutes)

Randomly assign groups and let students work on the rest of the activity. Circulate and check for accuracy and understanding.

We have allotted 30 minutes for group work. Ideally, all groups will complete #2-6 before you end group time. Faster groups can finish through #8, but not everyone needs to do #7 and #8.

Potential areas of difficulty:
- #2 Make sure that students are drawing the number line with a consistent scale. The boxplot should show a strong left skew.
- #4 Students may be stressing out about finding the exact value of the median and quartiles in the histogram. Encourage them to look at the medians in the boxplots and then use the median as a typical value when looking at the histogram. For example, the median of A is 1,

which histogram does NOT have a typical value of 1? (III). Thinking about shape is also helpful. Boxplot A does not have a left tail. What does this tell us? Which histogram (I or II) looks like it might have 25% of the data with a value of 0?

- In #5, students may have trouble getting started. Tell them to try something! You might even make up a data set that doesn't work and have them try to edit it to get closer to fitting the boxplot. The idea is the help students move away from looking for perfect answer at the start. Exploring and tweaking wrong answers is a great method here.

Suggested closure (about 25 minutes)

<u>Whole Class Quiz</u> (about 15 mins)

A Whole Class Quiz is a low stakes classroom assessment technique that can help increase individual accountability. It is also a good way to check for understanding and surface common misconceptions. Try to keep it light and fun.

The class starts with a score of 10. Randomly call on students to answer. If an answer is wrong, call on another student at random to correct. For each wrong answer that is not corrected, deduct 0.5 points and explain the right answer. If the class score dips below an 8.0 or so, add opportunities for bonus points.

Some suggested questions:

- In #4 Boxplot A, what is the median?
- In #4 which of the 3 data sets has the largest median?
- True or false, the 3 data sets have the same minimum.
- True or false, the 3 data sets have the same maximum.
- In #4 boxplot B, what is the 5-number summary? (Answer: 0, 1, 2, 3, 5)
- In #4 boxplot B, which quartile contains the most data? (Answer: all quartiles contain the same amount of data.)

<u>Student presentations on #6</u> (10 mins)

Or

<u>Watch a movie and discuss</u> (10 mins)

This movie contrasts histograms and boxplots. It addresses some common misconceptions, such as the belief that "longer" quartiles contain more data. http://somup.com/cqVUfxnbtE (4:33 mins)
After showing the movie, give students a few minutes to talk with a neighbor about what they learned.

Discussion of Brainology

Learning goals:

- Define growth and fixed mindsets.
- Discuss the role of mindset in our explaining our willingness to take risks and learn from mistakes.

Estimated time (40-60 mins)

Suggested facilitation:

If you have been using group work a lot, try Speed-dating. Speed-dating gets everyone involved.

Fess up (0-20mins)
Ask students if they read Brainology. Students who did not read it should move to the back of class and read the article. Everyone else can Speed-date at the front of the class. If a large proportion of the class did not read it, give a 15-minute break to those who can show you that they answered questions or annotated the article and make everyone else stay in class to read.

Speed-dating (30 mins)

- Review the logistics of Speed-dating. See Summary of Interactive Strategies.
- Students should introduce themselves in each round if they don't know their "date." Remind them that both people should share during the date.
- Do 6 rounds @ 4-mins each.
- Project one question at a time.

Suggested questions:

Do not use the exact questions from the homework assignment. If you do, students will tend to read their answers to each other instead of having a conversation.

- Round 1: What is a growth mindset? How does someone with a growth mindset respond to challenges and feedback?

- Round 2: What is a fixed mindset? How does someone with a fixed mindset respond to challenges and feedback?

- Round 3: Can you have a fixed mindset about some things and a growth mindset about others? If so, give some examples. If not, explain why not.

- Round 4: Describe some of the studies Dweck conducted to investigate the impact of mindsets on students' performance in school.

- Round 5: What in the Brainology article resonated with your experiences? What surprised you?

- Round 6: A friend or classmate repeatedly says things like, “I can’t do math. I am not a math person.” He also leaves class when he gets frustrated and gives up easily on Canvas homework when things are not immediately making sense to him. What might you share about Dweck’s work on mindsets to encourage him to persevere?

Closure (10-15 mins)

Facilitate a brief class discussion. Here are some questions you can use:

- What in the Brainology article resonated with your experiences? What surprised you?
- As a math teacher, I often hear students say “I can’t do math. I am not a math person.” How might this mindset affect a student’s behavior and performance?
- Can a fixed mindset be changed into a growth mindset? If so, what are some things that could help this transformation?
- Should we continue to include Brainology in the Math 110/110S packet? Why or why not?

6.2 Quantifying Variability Relative to the Median

Learning Goal: Compare distributions from two or more groups
Specific Learning Objectives: Create and interpret boxplots to compare distributions

Estimated time: about 80 minutes.

Suggested set up (15 mins)

In our experience, when we ask students to compare two distributions, they will compare means or medians and stop there. They have difficulty determining whether the difference between two medians (or two means) is substantial given the variability in the data.
This movie addresses this issue (7:17 minutes) http://somup.com/cqnec4nFFp

After the movie, think-pair-share for a total of 5 minutes: what did they learn from the movie? (Don't worry if they are missing some big points. This is just to get them talking about comparing distributions.)

Implementation suggestions (50 mins)

Randomly assign groups of 4.

Before students begin work on the activity, point out that the 1b requires a written paragraph. If you have already shared the Fortune 500 Most Valued Skills, remind them that oral communication, listening and writing are in the top 10 skills sought by the Fortune 500 companies. This activity is an opportunity to practice all three.

Suggest that they brainstorm as a group first and make some notes before each person writes his/her own paragraph. In the brainstorming they should address these questions:

- What is their thesis? (Similar or is one larger?)
- What evidence will they cite in support of their thesis?

Groupwork on (1a) and (1b) (10 mins)
Everyone needs to have something written down for Speed-dating.

Speed-dating (10 minutes)
Do 3 rounds, each 3 minutes to compare paragraphs for 1(b) repeatedly.This will allow students to practice explaining their analysis and get additional ideas for how to use the data to support their thesis.

Individual revisions (5 mins)
At the end of 10 minutes of speed dating, students return to their seats to revise their paragraph for about 5 minutes.

Repeat the cycle using (1c)

Groupwork on (1c) (10 mins)
Speed-dating on (1c) (10 mins): 3 rounds @ 3-mins
Individual revisions on (1c) (5 mins)

Potential areas of difficulty:
Students will just make a bulleted list of observations, but they will have difficulty using the observations to support their thesis or to compare the two distributions.

Many students will compare means or medians and stop there. They have difficulty determining whether the difference between two medians (or two means) is substantial given the variability in the data. Prod them to compare the middle half of the data. Use the interval Q1 to Q3 to represent an interval of typical measurements. If these intervals overlap substantially, then the distributions are similar because typical measurements are similar. If these intervals do not overlap much, then one distribution has measurements that are typically larger than the other.

Closure (15 mins)

Discuss model answers. Students have to produce a similar analysis in 6.3 Lab. Project the model paragraphs one at a time. Ask a student to read it out loud and discuss what makes the paragraph good, e.g. clear thesis, not a list but a comparison of two groups, use of course concepts to support thesis, use of many course concepts.

Pelvis diameters:
In this sample, men and women have similar pelvic diameters. The median is about 28 cm for each sex. Both distributions are roughly symmetric with an overall range of about 15 cm. Typical males, as measured by the interval Q1 to Q3, vary between 26.8 cm and 29.5 cm in their pelvic measurements, compared to a very similar interval for typical females, who vary from 26.2 to 29.2 cm. This tells us that the middle half of each distribution have similar pelvic measurements.

Wrist girths:
In this sample, the distributions of wrist girths are symmetric with similar ranges: men (5.2 cm) vs. women (5 cm). Despite these similarities in shape and overall spread, men have substantially larger wrist girths than women. The median wrist measurement for men is 17.1 cm compared to 15 cm for women. This 2.1 cm difference is large in light of the variability in the middle half of each distribution (IQR). For each sex, the IQR is less than this difference: men (1.4 cm) vs. women (1.1 cm.) As a result, the typical measurements as represented by the interval Q1 to Q3 do not overlap. The wrists of typical men vary between 16.5 to 17.9 cm; very few women have wrist measurements this large. The wrists of typical women are smaller and vary from 14.5 cm to 15.6 cm; very few men have wrist measurements that are this small.

7.1 Measuring Variability Relative to the Mean

Learning Objective: Distinguish between graphs with small or large standard deviation using the concept of average deviation from the mean.

Estimated time: 100 minutes

Suggested set up (about 30 minutes)

Work through 1-3 interactively. Here are some suggestions:

For #1 (about 15 minutes):

- Students may have difficulty understanding the variable (average monthly high temperature over 10 years) and the individual (a month). To help clarify the variable, you might need to sketch a quick spreadsheet with row labels as months and column labels as 2002, 2001, … 2012, and average monthly temperature.

- Do an iterative series of 3-minute Think (30 seconds)–Pair (30 seconds)–Share (1 min)
    - During Share, randomly choose students to answer, to add to someone's answer, or to respectfully correct by providing a different perspective.
    - Repeat for each part of #1.
    - For 1b) and 1c) the limited amount of time will discourage calculations. We do not want students calculating here; instead we want them to focus on variability. The goal is to reinforce the concept of variability about the median and to make the transition to variability about the mean.

For #2 and the table (about 15 minutes)

- Get someone to read the paragraph above #2 to clarify the goal for the rest of the activity.
- The questions in #2 are pretty easy for students. You can continue with faster iterative Think (30 seconds)–Pair (30 seconds)–Share (30 seconds) for each question.
- Have someone read the two sentences about standard deviation.
- Think (2 mins)–Pair (2 mins)–Share (4 mins) to complete the table and find the average distance that scores vary from the mean.
    - A common mistake in calculating the average distance is to ignore the points at the mean. Here students may erroneously divide by 8 (and get an average distance of 20) instead of dividing by 11 (to get an average distance of about 14.5). If this occurs, surface the issue (Should we divide by 8 or 11?) and emphasize that we have to include all of the data.
    - This issue is reinforced in #3, so don't worry if students seem unsure at this point. Move on to #3.

For #3 (about 5 minutes)

- Another Think (30 seconds)–Pair (30 seconds)–Share (2 mins) will work here but figure out a way to discourage calculations. Maybe have students hold their hands in the air for the 1-minute Think-Pair portion (or stand and put their hands on their hips while they think and discuss with a neighbor.)

- For the discussion, quickly tally their opinions (larger, smaller, same) and get students to provide explanations.
    - Ultimately, what we want them to understand is this: relative to the distribution in #2, the distribution in #3 is more tightly grouped about the mean; there is a much larger proportion of data at the mean. Therefore, the average distance of all of the data from the mean will be smaller.
    - Verify this by quickly calculating the average distance from the mean (ADM~9.1)

Suggested implementation (about 30 minutes)

- Randomly assign groups.
- Have groups work on #4-9.
- Note: #10 is really challenging for students; it involves finding counter examples to that challenge common misconceptions. Therefore, we suggest that you skip #10 and instead integrate the common misconceptions into the discussion of #4-#8 as outlined under Closure.
- For faster groups, assign them a problem to present and/or assign them the role of Roving Tutor.

Potential areas of difficulty:

- For #4-7, many will ignore the instructions and calculate ADM because we did this in the set-up. As you circulate, call their attention to these instructions. Ask groups to predict which data set for a given problem will have the smallest ADM before they do any calculations. Alternatively, if they have already completed calculations, ask students why it makes sense that a given data set has a smaller ADM than the other. (We want them to see that a larger proportion of the data is closer to the mean.)

- One common mistake that may continue into groupwork is the issue of what to do with points at the mean. Some students may continue to ignore points at the mean in their tally since the distance from the mean is zero. In other words, they will divide by the wrong total.

- For #8, students may be unsure about the location of the mean. Remind them that the mean is the balancing point; or, alternatively, point out that the distributions are symmetric, so the mean is the same as the median and located in the center.

- They may also add numbers to the horizontal axis and compute the ADM. If this is happening, suggest that they change the scale (e.g. count by 5's), and ask if it will change their answer. Why or why not? What if the scale is 10?

Suggested Closure (about 40 minutes)

Here are several ideas for closure. Choose one or several to clarify concepts and address issues that you observed during group work.

<u>Student presentations of #4-8</u> (35 minutes)

Randomly choose a group to present each problem for #4-#8. Let students use the document camera to save time. This is an opportunity for students to gain confidence presenting.

- If students calculate ADM, you can check their answers using the table in #10.
- In the discussion of each problem, focus on how we can <u>visually</u> identify the dot plot with smallest ADM.
- After the presentation on #5, discuss some common misconceptions:

- Misconception: When comparing two data sets, the data set with the fewest data points will have the ADM.

    - Class C has fewer students but a larger ADM. Why does this make sense? [The data varies more about the mean and there are fewer data points at the mean.]

- Misconception: When comparing two data sets, the dot plot with the shortest stacks of data (smallest frequencies) will have the smaller ADM.

    - Class C has short stacks of data but the larger ADM. What does this make sense? [Frequency of each data value is smaller, but variability is larger. There is more data further from the mean.]

- After the presentation on #6, discuss this common misconception: When comparing two data sets, the one with the smaller overall range always has the smaller ADM.

<u>Ambassador exchange</u> (about 20 mins)

- (7 mins) Ambassadors compare answers with a host group with the goal of checking work and getting additional thoughts on a problem that was difficult for the home group.
- (7 mins) Ambassadors return. Groups discuss what they learned and resolve any lingering issues.
- (6 mins) As a class discuss any lingering issues that were not resolved during the Ambassador Exchange.

Whole class quiz (15 mins)

Quiz the class on the answers and add extension problems. Here are some suggestions:

- In #4 which has smallest ADM?
- Describe another data set that would have the same ADM as Class A and B.
- In #5 which has smallest ADM?
- Which class has the smaller range, C or D?
- Identify the data point that is the furthest from the mean in Class D.
- In #6 which has the smallest ADM?
- Which class has the smaller range, E or F?
- Draw the dotplot for #7.
- Which has the smallest ADM: {70,70,70} or {70,70,70,70,70}?

Think (2 mins)–Pair (2 mins)–Share (4 mins) about some common misconceptions (25 mins)

- Misconception: When comparing two data sets, the data set with the fewest data points will have the smallest ADM.
  Make a counter-example by constructing two distributions so that the distribution with more data points has a smaller ADM.

- Misconception: When comparing two data sets, the dot plot with the shortest stacks of data (smallest frequencies) will have the smaller ADM.
  Make a counter-example by constructing two distributions so that the distribution with the shortest stacks has the larger ADM.

- Misconception: When comparing two distributions, the distribution with the larger range will have the larger ADM.
  Make a counter-example by constructing two distributions so that the distribution with a larger range has the smaller ADM.

Discussion of #9 (5 minutes)
Call on some students to share what they wrote for #9.

Discussion of You Can Grow Your Brain

Learning goal: Explain how this course is designed to grow your brain.

Estimated time: 20 mins

Suggested facilitation:

If you have been using group work a lot, try Speed-dating. Speed-dating gets everyone involved.

<u>Fess up</u> (2 mins)
Ask students if they read You Can Grow Your Brain. Students who did not read it should move to the back of class and read the article. Everyone else can Speed-date at the front of the class.

<u>Speed-dating</u> (15 mins)

- Review the logistics of Speed-dating. See Summary of Interactive Strategies.
- Students should introduce themselves in each round if they don't know their "date." Remind them that both people should share during the date.
- Do 4 rounds @ 3-mins each.
- Project one question at a time.

Suggested questions:

Round 1: How is the brain like a muscle?

Round 2: Can you literally grow your brain or is this just a metaphor? Explain.

Round 3: What is the formula for growing the brain?

Round 4: How does this course grow your brain? Be specific.

Is the following statement true or false? Use what you learned from the article to support your answer. Statement: You are born with a certain amount of intelligence and this is fixed for life.

Closure (5 mins)

Discuss this question: Should we continue to include You Can Grow Your Brain in the Math 110/110S packet? Why or why not?

7.2 Using the ADM

Learning objective: Use the concept of average deviation from the mean to estimate the standard deviation from the mean.

Standard deviation is a very difficult concept for students to understand. In fact, research shows that standard deviation questions are frequently missed on comprehensive introductory statistics exams in national samples of students who have completed introductory statistics courses. (http://iase-web.org/documents/SERJ/SERJ6(2)_delMas.pdf) For this reason, we continue to build toward an understanding of standard deviation using the more intuitive ADM.

In this activity students are not doing much calculating. Instead they mainly comparing distributions to pick the one with the larger or smaller average deviation about the mean. This builds intuition and understanding of variability about the mean. In 7.3 we will introduce standard deviation and perform a few calculations.

Estimated time: about 65 minutes

Suggested set-up (about 20 minutes)

Discuss (1a) and (1b) using quick Think (30 seconds)–Pair (30 seconds)–Share (2 mins) to discourage calculations.

Discuss (1c): Think (2 min)– Pair (2 min): Divide the class into thirds and assign each section one dotplot. Think–Pair to calculate ADM for one dotplot.

Presentations on (1c) (9 mins): Ask for volunteers to present.

Implementation (about 35 minutes)

Students work on #2-#7 in randomly assigned groups.

Potential areas of difficulty:

#4: Some groups may get stuck on #4. They may be troubled that they cannot calculate the mean or the ADM. Point out that we do not have to give the ADMs; we just have to identify which distribution has a larger ADM.

- Ask students to estimate the location of the mean. Then ask which histogram has a larger proportion of data near the mean.
- Alternatively, suggest that they translate the histograms into reasonable dot plots or lists.

#5-#7 Encourage students to try something. You might even make up a small data set with a mean that is a whole number and see if students can figure out how to make up another data set with the same mean that has a different ADM. They can do this by moving data points symmetrically about the mean. Once they get going with a guess and check method for #5, they will probably be OK with #6-7.

Closure (about 10 minutes)

Class discussion

- Ask each group to pick a problem they would like to hear discussed. Let students present solutions at the board for the most popular problems.

- Be sure to ask the class if everyone agrees with what has been proposed as a solution. If someone ends up correcting a student's idea or work, give the original author kudos for starting us off on a good discussion.

7.3 The Standard Deviation

Learning Goal: For the distribution of a quantitative variable, describe the overall pattern (shape, center, and spread) and striking deviations from the pattern.

Specific Learning Objective: Estimate and calculate the standard deviation.

Total estimated time: 60 minutes

We will not require students to calculate SD by hand in Canvas exercises, but we do one example in this activity. After this, we will use StatCrunch to calculate SD.

Suggested Set Up (25 minutes)

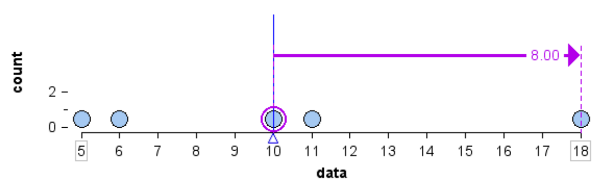
Discuss Warm-up problems:

NOTE: We recommend introducing the formulas in (2) AFTER calculating ADM in (1) as a way to symbolically summarize what they already understand. We have found that this builds better conceptual understanding than the traditional approach of starting with the formula.

<u>Discuss (1)</u> (about 5 mins)

Ask students to find the mean (x-bar=10) and to help fill in the table. For each entry in the table, indicate the distance on the dotplot as shown below.

Data value	5	6	10	11	18
Distance from the mean					



Now find the average distance from the mean. Sum the distances, then divide by n=5 (ADM=3.6)

<u>Discuss (2)</u> (about 10 mins)

(5 mins) Discuss the formula for ADM first. Define x, x-bar and n and explain how the formula summarizes what we just did. Something like this …

- ◊ We found the distance of each data value from the mean; this is $|x - \bar{x}|$.
- ◊ Why do we use the absolute value? Remind them that if we add signed deviations from the mean, we always get 0. Reinforce notation by summarizing this as $\sum(x - \bar{x}) = 0$ ALWAYS! If we always get 0, we aren't measuring anything.
- ◊ We then added up these distances; this is $\sum|x - \bar{x}|$.

◊ Then we found the average distance by dividing by the number of data values; this is $\frac{\sum|x-\bar{x}|}{n}$.

Ask a few questions to help students understand the formula, such as

◊ What is n for our data set?
◊ What is x-bar?
◊ For the data value at 6, what is the value of $|x - \bar{x}|$? What does this value tell us?
◊ What part of the formula tells us to add up the distances from the mean?
◊ How can you tell by looking at the formula that this is an average of distances from the mean?

(5 mins) Ask students to compare the ADM and SD formulas

They should notice the following (if they don't, ask questions to elicit these observations):
- Both formulas involve the mean (x-bar)
- Both involve calculating deviation from the mean (x minus x-bar)
- Both involve a way to make $x - \bar{x}$ positive.
- Both involve an averaging (summing and then dividing by n or n-1).

Potential area of difficulty: Students are likely to ask why SD involves dividing by n-1, rather than n. This can be tricky to explain. Here is our suggestion: Actually, both n and n-1 are used to calculate standard deviation. However, in college-level Statistics, we are usually calculating the standard deviation of a sample. In this case, the formula with n–1 is used. Why? Because in college Statistics we use data from a sample to estimate something about a larger population. SD calculated with n-1 using data values from a sample is a better way to estimate the SD in the larger population. (Note to instructors: dividing by n-1 makes the SD an unbiased estimator of the population SD, i.e. the mean of the sample SDs is the population SD, though this is probably too complicated to share with students.)

Discuss (3) (about 10 minutes)

- Demonstrate the calculation of SD.
- Reassure students that we will be using technology to calculate SD. This demonstration is to help them understand the formula. None of the problems in the activity packet or in OLI will require by-hand calculations of SD.
- Represent the SD graphically by drawing a SD hatplot above the dotplot.
  - Mark the mean. The box has endpoints at one SD above and below the mean.
  - The box marks off a typical interval of values, much like the idea of using Q1 to Q3 for identifying an interval of typical values.

Implementation (about 20 minutes)

Divide students into random groups of 4 to work on #4-9. Some of these problems are similar (and even identical) to problems they already answered using ADM. This is intentional. We want students to think of SD as approximately equal to ADM.

If a group finishes early, here are some additional problems that they can do:

1. Make up a data set of 5 numbers using 0 to 20 that has a mean of 10 and a larger SD than the data set in #1. Explain your strategy then calculate the SD to show that you got it right.

2. Make up another data set of 5 numbers using 0 to 20 that has a mean of 10 and a smaller SD then the data set in #1. Explain your strategy then calculate the SD to show that you got it right.

Potential areas of difficulty:

- #6: Some students think the uniform distribution of problem 4 has a smaller SD because it's the flatter of the two. To troubleshoot this, remind students that ADM is a good estimate for SD; which has a smaller ADM?

  Alternatively, ask students to estimate the location of the mean (around 6), then ask which histogram has a larger proportion of data near the mean.

- #7: Students may think that the answer is (a). It is true that if everyone made a 100%, the SD =0. Ask them if there are other possibilities.

Closure (15 minutes)

Here are some ideas for closing this activity. At the very end of the lesson, ask students to share their descriptions of what SD measures (#9).

Ambassador exchange, followed by class discussion to clear up lingering issues.

Since the answers are right or wrong, an Ambassador Exchange is an easy way for groups to check their answers and identify areas that may need more discussion.

Challenge round: Use Think–Pair–Share on these challenging questions:

- Who can come up with a set of 5 numbers (using whole numbers $0 \leq X \leq 20$) with a mean of 10 and the smallest SD possible?
- Who can come up with a set of 5 numbers (using whole numbers $0 \leq X \leq 20$) with a mean of 10 and the largest SD possible?

7.4 The Mean and Standard Deviation: Intervals of Typical Measurements

Learning Goal: For the distribution of a quantitative variable, describe the overall pattern (shape, center, and spread) and striking deviations from the pattern.

Specific Learning Objective: Use the mean and standard deviation to create intervals of typical measurements.

Estimated time: 55 minutes

Suggest Set Up (about 20 minutes)

Ask a student to read the paragraph after the specific learning goals for the class. Facilitate a class discussion of what the paragraph says.

Alternatively, consider starting with a "big picture" summary of key concepts that relate to this lesson. For this lesson, you say something like this: Statisticians describe a distribution using shape, center and spread.

- ◊ We think of the "center" as a single value that represents the distribution; it can be viewed as a typical value. We started off by eyeballing a value to describe the "center" by looking at a dotplot of the data. Then we moved to using mean or median as options for describing "center".
- ◊ Instead of just using one number to summarize a distribution (mean or median), we also took variability into account and used an interval of values to describe typical data values. Typical values can be described by the interval Q1 to Q3, this is the middle half of the data. Or we can use the ADM or the SD to find a typical interval about the mean. This is our focus for today.

Work through #1-#4 interactively with the class. You can again use quick iterative cycles of Think-Pair-Share, tackling one question at a time. During the Share, call on students randomly to answer the questions. Note: Random selection increases engagement. If a student is unable to answer, she can confer with her partner and then answer, or you can return to her later to repeat an explanation or to answer another question.

For the most part, students are usually able to answer many of these questions, but there still may be some confusion about the significance of overlapping intervals of typical values and how to use a hatplot to estimate SD.

Suggested implementation (about 20 minutes)

Randomly assign groups of 4 to do #5-6. As you circulate, check for answers for accuracy. Flag problematic areas for each group and have them revisit their answers.

Suggested Closure (about 15 minutes)

Here are several ideas. You may only have time for one.

Whole-class low stakes quiz:

Treat #5-6b as the quiz. Think of some kind of reward if the class gets a 9 out of 10 or better, e.g. candy, bonus points, a longer break next class.

Here is how the whole-class quiz works:

- ◊ Randomly select a student.
- ◊ If the student works the problem correctly, move to the next problem
- ◊ If the first student declines to work the problem, deduct 0.5 points.
- ◊ If he does not work the problem correctly, randomly select a second student to identify and correct any mistakes.
- ◊ If the second student does not fix all of the mistakes, deduct another half of a point, and do the problem for the class.
- ◊ Repeat until the quiz is finished.

Ambassador Exchange:

With an Ambassador Exchange, groups check each other's work on #5-6b. Clear up any lingering issues after the end of the Ambassador Exchange.

Group shuffle to identify strongest answers to (6c):

- Quickly shuffle the groups.
- New groups compare answers to (6c) and identify the one they like best or meld several answers together into a stronger response.
- Randomly chose a few groups to share on the document camera or ask for a few volunteers.

7.7 Unit 2 Project

Estimated time: 70 minutes

Run the Unit 2 Project like a "conference style" poster session. This project requires the use of StatCrunch

Below are general notes for facilitating "Conference style" Poster Presentations

Overview of how to run a "conference style" poster presentations

- Students work in groups on a difficult problem that has multiple acceptable answers or multiple solution strategies.
- Each group makes a poster to summarize their analysis and conclusion.
- One group member stays at the poster to present the findings.
- The rest of the group rotates from poster-to-poster to hear short presentations, similar to a gallery walk for a poster session at a conference.
- Students who are not presenting have a listening/responding protocol that fosters active engagement during each presentation.

What materials are needed?

- Access to StatCrunch
- Access to the data through our StatCrunch group (Math 110 Los Medanos College)
- Graph paper and plain notebook paper (2-3 sheets per group)
- Poster paper (one to two sheets for each group of 4 students)
- Tape
- Markers (1-2 for each group of 4 students)
- Post-its (3x3in, two colors; for a class of 32, 168 post-its in each color)

How do I facilitate a poster session and gallery walk?

<u>Set-up:</u> Approximately 5 minutes.
Review the instructions. Explain how to access the data sets.

<u>Group work</u>: Approximately 25 minutes.

Students work in groups of 4 to answer the question using observations from the data. Each group makes a poster with graphs, diagrams, tables, verbal descriptions, and/or calculations from their StatCrunch analysis.

All students should be engaged in the making of the poster, e.g. some can translate StatCrunch graphs onto graph paper, others can write the verbal descriptions on notebook paper, etc. These parts can be taped to the poster.

Hang posters so that the four problems occur in a side-by-side sequence, followed by another sequence of 4 posters representing the 4 problems.

Poster presentations: Approximately 20 minutes
Dry run: 5 minutes for groups to practice prior to the gallery walk;
Gallery walk: four 3-min rounds (rotating group gets to hear presentations on each of the 4 problems.)

Instructions to students:

- One student will be the presenter. (The instructor can choose the presenter using random assignment or choose someone who looks disengaged or lost.).
- Groups have 3-5 minutes to do a dry run. This gives the presenter time to practice and get feedback from the group.
- Groups will rotate together (minus the presenter) when the lights flicker (or alarm sounds). Each round will last about 3 minutes.
- Before rotations begin, give stacks of the two colors of post-its to each presenter.
- Each student listening to a presentation will provide feedback to each presenter on post-its. One color is for a kudo: a positive note that captures something that was clear or compelling in the analysis. Another color is for a piece of constructive feedback: a note that highlights something that was unclear or perhaps incorrect or something that could be expanded in the analysis. Each student must leave two post-its (one of each type) at each poster.

Note: With 4 rotations, each presenter will do his/her presentation 4 times. This allows the presenters the opportunity to improve their understanding of their group's analysis and to improve their explanation in a low-stakes environment. This is a reason to pick a student who seems disengaged or lost.

Note: Insist that each student leaves both a kudo and a piece of constructive feedback at each poster. This helps students learn to mentally engage with a presentation, to treat their peers' ideas with respect, and to learn to disagree in positive and constructive ways. Monitor this during the gallery walk by visually checking to see if there is an equal number of post-its in the two colors at each poster and that are each round, the number of post-its increases by 6 (3 of each color, left by each student.)

Closure: Approximately 20 minutes.

(5 mins) Review feedback: When the groups have rotated back to their own poster, they will naturally begin to look at the feedback on the post-its. They will usually also begin to explain what was on other posters to the presenter, who will probably be asking about this.

(15 mins) Discussion

- Ask students to return to their seats for discussion.

- For each Option, have the class compare and contrast the posters for that Option. What is the answer to the question? Did the groups agree? What did you find the compelling or convincing in the analysis?

- After the discussion, switch the conversation to feedback. Ask each presenter to share the most helpful piece of feedback that they received. Emphasize that feedback is an invaluable way to improve your work and that it is a fundamental part of working productively in teams.

Follow-up assignment: Consider creating an assignment where each student answers the question of their choice in writing with support of the data. The purpose of this assignment is increase accountability in group work, motivate students to learn from each other, and communicate that work on real life problems takes time and effort. We will use this cycle of brainstorming, presentation of preliminary ideas, feedback, revision throughout the many projects we do in this course.

8.1 Introduction to Scatterplots and Association

Learning Goal: Use a scatterplot to display the relationship between two quantitative variables. Describe the overall pattern and striking deviations from the pattern.

Specific Learning Objectives:

- Read and interpret scatterplots
- Describe the pattern in a scatterplot as positive or negative association, if appropriate.

Estimated time: 80 minutes

Suggestions for Set Up (About 5 minutes)

This activity does not need much set-up. Students are usually somewhat familiar with scatterplots and are able to figure out the answers to the questions in this activity without much help.

We do not recommend spending class time on making a scatterplot by hand. This activity focuses on reading and interpreting scatterplots and recognizing patterns. You also do not need to talk about explanatory and response variables here. This is covered in the next activity.

- Focus the class by asking someone to read the Overview.
- Then ask a few questions to connect the values in the spreadsheet to the scatterplot. Project the scatterplot using the document camera. Use Think-Pair-Share and randomly call on students to locate the dot using the document camera.
  - Where is the first person in the spreadsheet located in the scatterplot? How do you know?
  - Where is the last person in the spreadsheet located? How do you know?

Suggestions for implementation (About 45 minutes)

Randomly assign groups of 4. Our experience is that students do well on this activity without much support.

Circulate to monitor students' work. All of the questions have a right or wrong answer, so let students know when you see an error so that they can figure out how to fix it.

If you are having trouble with groups staying focused during group work, consider doing a low stakes Whole Class Quiz at the end of the lesson. Tell them now that you plan to do this. What will the reward be for good class performance, e.g. candy, a longer break, bonus points? If you haven't already done a Whole Class Quiz, explain how it works. A Whole Class Quiz can increase motivation and accountability during group work … and if executed well, it can be fun!

Suggested Closure (About 30 minutes)

Here are several ideas for closure. Pick one or do several if you have time.

Discuss #11 and #12

Speed-dating

Since the answers are right or wrong, Speed-dating allows every to check their answers and increases individual accountability for what transpired during group work.

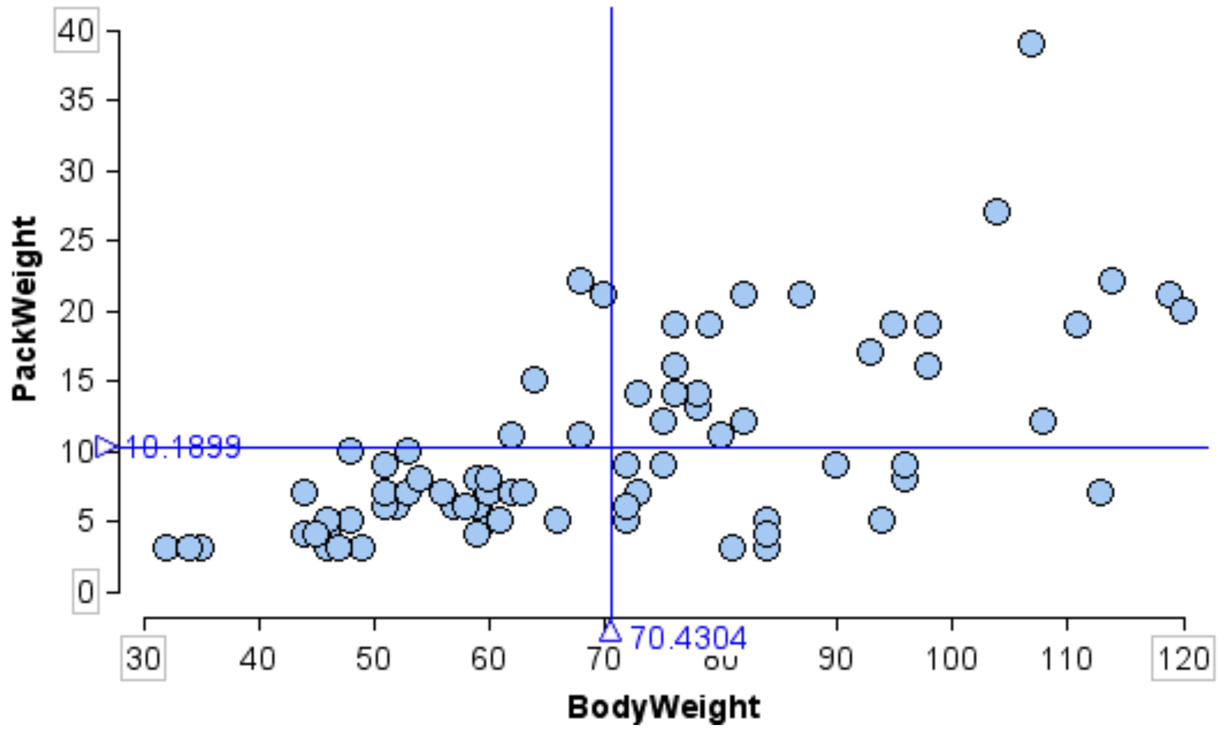
Do one round on each question. Short rounds will work here (1- to 1.5-mins)

Do a low stakes Whole Class Quiz

Use some subset of the problems from the activity OR use similar questions in a new context, e.g. Body weight (lbs) vs. Backpack weight (lbs) for elementary school children. See scatterplot and sample questions provided below.

See Summary of Interactive Strategies if you need a reminder about how a Whole Class Quiz works.

Body weight (lbs) vs. Backpack weight (lbs) for a sample of elementary school students.



Sample questions:

1) Approximately much does the student weigh who is carrying the heaviest backpack?
2) What can we say about the backpack weights for students weighing between 30 and 40 pounds?
3) What does the 70.4304 represent?
4) What does the 10.1899 represent?
5) Circle a student who is lighter than average but who carries a heavier than average backpack.
6) Circle a student who is heavier than average but is carrying a lighter than average backpack.
7) Is the association between body weight and backpack weight positive or negative? How does the graph help you decide?
8) Draw a scatterplot with at least 5 data points that has a negative association.
9) Would the following tend to have a positive or negative association? Why do you think so? [both tend to be negatively associated)
    - Miles above sea level at a national park vs. temperature
    - Weight of a car vs. average miles per gallon on the highway

Module 8.2 Direction, Form and Strength in Scatterplots

Learning Goal: Use a scatterplot to display the relationship between two quantitative variables. Describe the overall pattern and striking deviations from the pattern.

Estimated time: 70 minutes

Specific Learning Objectives:

- Identify explanatory and response variables.
- Read and interpret scatterplots.
- Identify direction, strength and form in scatterplots.

Big Ideas:

- When we use a scatterplot, the goal is to look at the relationship between two quantitative variables.
- To determine which variable goes on the horizontal axis vs. vertical axis, we need to determine which variable is the outcome (response) and which might be affecting that outcome (explanatory).
- With a scatterplot, we describe direction (positive vs. negative), strength (weak to strong), and form (linear or nonlinear). This is similar to describing shape (direction and form), center and spread (strength) with a dotplot.

Suggested set up (20 min)

<u>Quick review</u>

Project a scatterplot of body weight vs. pack weight for a sample of elementary school children given at the end of the 8.2 facilitation notes.

Think (30 seconds)–Pair (30 seconds)–Share (1 min) with the following questions:

- ◊ Who are the individuals?
- ◊ What are the variables?
- ◊ Are the variables categorical or quantitative?
- ◊ Is the association positive or negative? How can we tell?

In this activity we will learn additional ways to describe the distribution in a scatterplot.

<u>Discuss explanatory and response variables</u>

- Ask a student to read the definitions of explanatory and response variables from the activity.
- Explain why body weight is explanatory and pack weight is response. (One way to explain this is to think about making predictions. We could use body weight to determine a safe

backpack weight. If we had a formula to give parents, body weight would be the input and a safe pack weight would be the output.)

Discuss (1) as a class

Use Think (30 seconds)–Pair (30 seconds)–Share (1-2 mins). Do one at a time for 3 rounds of Think-Pair-Share.

If students are having difficulty identifying the variables, try making a mock spreadsheet for each one. For example, for (1a) each row represents a student; the 1st column is homework average; the second column is exam average. For (1c) each row represents a person at the bar; the 1st column is BAC; 2nd column is number of alcoholic drinks.

Note: students may tend to think of the explanatory variable as CAUSING a change in the response. In a later activity we will tackle the correlation vs. causation issue. Don't worry about it now.

Suggested Implementation (40 min)

Randomly assign students to groups of 4 to work on #2-#5.

Students may find this activity somewhat challenging because there are no units on the axes. We removed the units to force them to use the direction, strength and form to make decisions about which scatterplots are plausible representations of the relationships described.

As you circulate, ask students to explain their reasoning (both for questions they have answered correctly and incorrectly.) Since there are right answers to these questions, don't hesitate to ask students to rethink wrong answers.

Potential areas of difficulty:

#2b: If students are struggling here, ask which piece of data could be collected first (opening box office sales) to predict a latter occurrence (Oscar win). Thinking about the timing can help identify explanatory (opening box office) vs. response (Oscar win).

#3: If students are struggling, try the following:

- Ask students to label the scatterplots as positive or negative association.
- Then go through each option in (3b) and label the association as positive or negative.
- Then go back to (3a) and match descriptions.
- Students may have difficulty with the city mpg vs. highway mpg. They may be thinking that for an individual vehicle, city mpg will be lower than highway mpg, so the association should be negative.

  Ask them to think about a large gas-guzzling vehicle like a Hummer and a small fuelefficient car like a Toyota Hybrid. The Hummer will get low mpg in both situations, while the Toyota gets high mpg in both situations. Of course, city mpg may be lower than highway mpg for both vehicles but this does not affect the direction of the association. Individual cars with low mpg in city will have fairly low mpg on highway (e.g. Hummer). Similarly, cars with higher mpg in city will have fairly high mpg on highway (e.g. Toyota Hybrid).

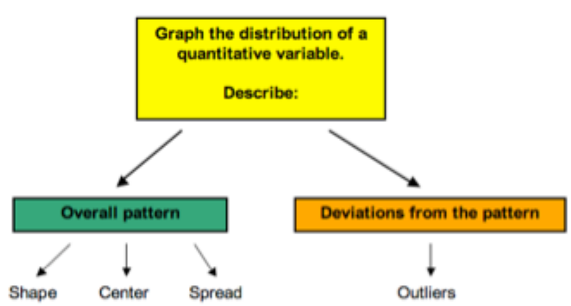
Suggested Closure (10 min)

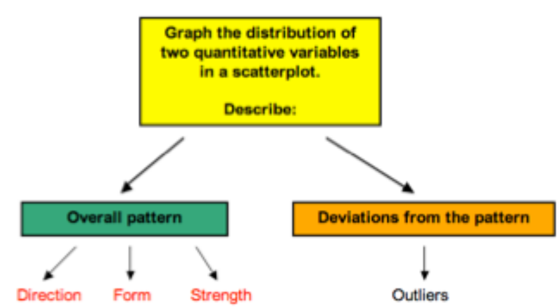
Project the body weight vs. pack weight scatterplot with the categorical variable pain in color given at the end of the 8.2 facilitation notes.

Ask students to compare the students with pain (purple) to the students without pain (green) in the scatterplot by asking the following questions:

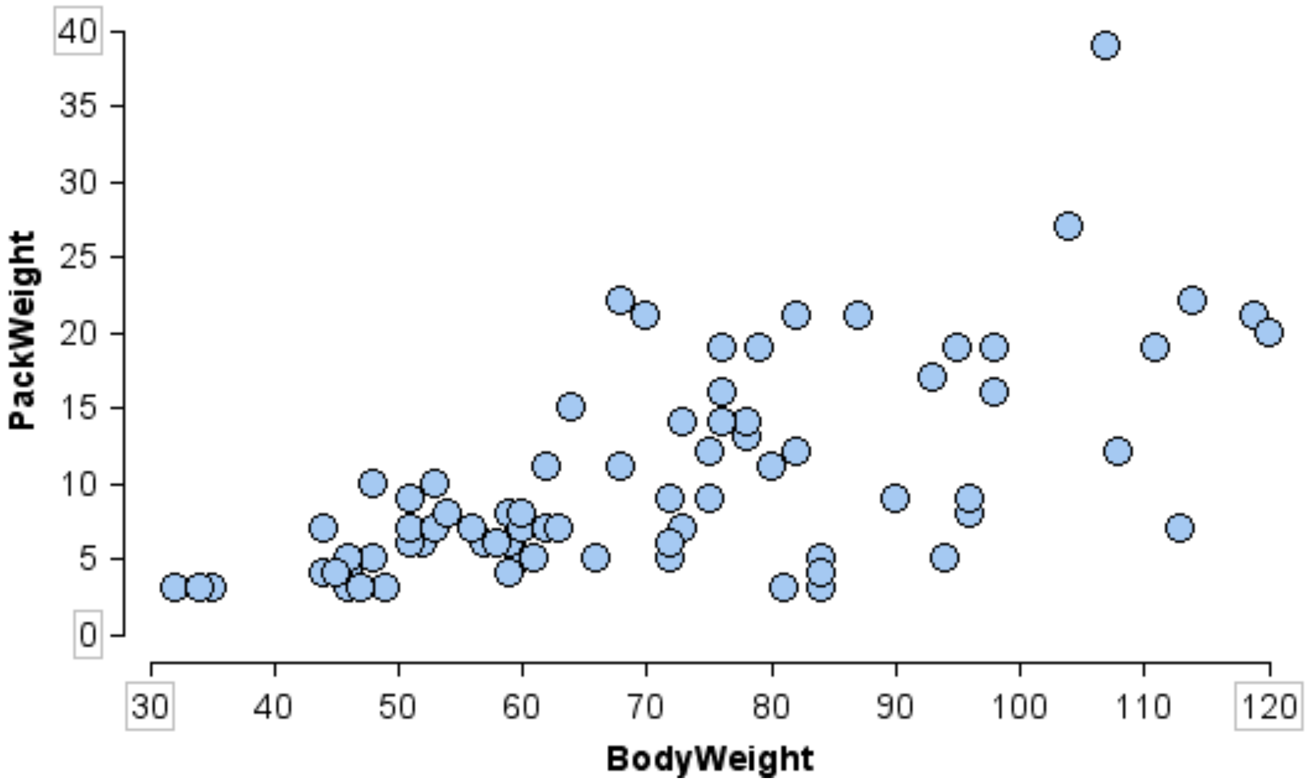
- ◊ What is the direction of the association between body weight and pack weight for each group? What does this tell us?
- ◊ Is the association stronger for one group? If so, which one? How can you tell?
- ◊ Does either group have an association that is fairly linear?

Now summarize the activity by comparing the way we describe dotplots to the way we describe scatterplots. Put the following two diagrams on the board and discuss them. For example, you can ask students to provide words used to describe shape, center and spread, as well as direction, form and strength.



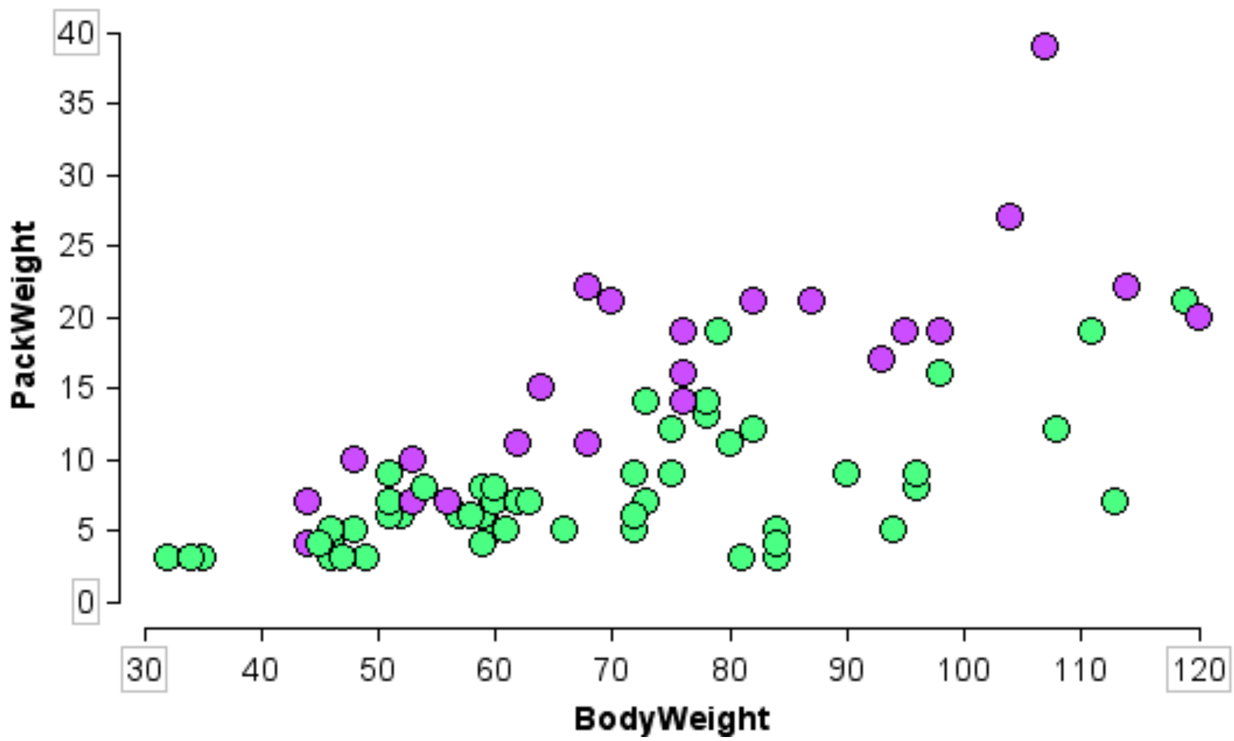


Body weight (lbs) vs. Backpack weight (lbs) for a sample of elementary school students



Body weight (lbs) vs. Backpack weight (lbs)
for a sample of elementary school students

- **Purple: students reporting back pain**
- **Green: students with no reported back pain**



Module 8.3 Introduction to Linear Correlation: r

Learning Goal: Use a scatterplot to display the relationship between two quantitative variables. Describe the overall pattern and striking deviations from the pattern.

Learning Objectives: Use r to describe strength and direction for a linear relationship.

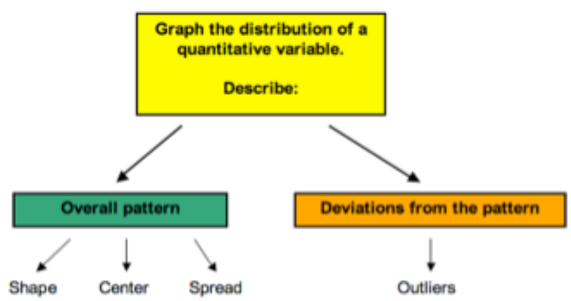
Estimated time: 35 minutes

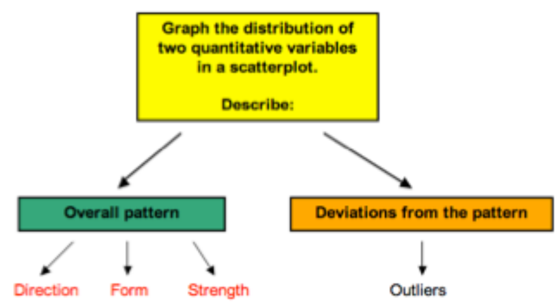
Big Ideas:

- The correlation coefficient, r, gives us information about the strength and direction of a linear relationship.
- Correlation values vary between -1 and 1, with ±1 a perfect linear relationship.
- Correlation is weakest near 0.

Suggested set up (5 min)

Put the following diagrams on the board or pull up this page in Canvas.





Ask students to recall the different ways we measure center (mean and median) and spread (range, IQR, ADM, SD). Remind them that these measures are useful when we have a distribution of a single quantitative variable, such as weight of a back pack or sugar in a serving of cereal. These measures go with dotplots, histograms or boxplots.

In this activity, we will focus on scatterplots with a linear form. We will develop a more precise way to measure direction and strength of linear relationships. This new numerical measure is called the correlation coefficient, which is abbreviated "r". By the end of this activity, you will know what r measures.

Ask a student to read the introduction to the activity. To call their attention to the r-values, tell them to write "r=0.94" under Scatterplot 1, "r=1" under Scatterplot 2, etc.

Suggested implementation (20 min)

Randomly assign groups.

Our experience is that all groups will realize that a positive association has a positive r-value and a negative association has a negative r-value.

Most groups will discover the "big ideas" on their own.

If a group is not keying into the "big ideas", here are some questions you can ask:

- ◊ For a minute, ignore the scatterplots and just look at the numbers in the table, what do you notice about these numbers? (All fall between -1 and 1.)
- ◊ What does the scatterplot look like when r=1 or -1?
- ◊ Which scatterplot has an r-value that is closest to 1, but not equal to 1? What do you notice about that scatterplot?
- ◊ Which scatterplot has an r-value that is closest to -1, but not equal to -1? What do you notice about that scatterplot?
- ◊ Which r-value is the closest to 0? What do you notice about that scatterplot?
- ◊ See if they can now make some generalizations about r. Give them time. If they can't, ask them to complete this sentence, "When r is equal to 1, the scatterplot ….". Make up similar sentences for them to complete for r close to 1, etc.

Students may confuse r with slope. Don't bring up the concept of slope unless students ask about it. (We discuss slope in depth in later activities.) If students are making statements about slope, ask, "How is slope related to the r-value? Which two scatterplots have the same r-value but different slopes? Can you draw another scatterplot with the same r-value but an even steeper slope?"

Suggested closure (10 min)

Go over #3 as a class. Call on students to share what they wrote for *#3a* to generate the "big ideas". Ask a few students to explain why they think r can never be bigger than 1 or less than -1. This may be difficult to verbalize, so let a few try it then reinforce the point that r is bounded between ±1.

After you get a good summary of "big ideas", go back to the previous activity 8.2.

- For #3. Ask students to match r-values to scatterplots. Here are the three plausible r-values for those three scatterplots (scrambled): 0.75, 0.15, -0.70.
- Repeat for #4. Plausible r-values (scrambled): 0.09, 0.65, 0.90

8.4 Correlation is not causation

Learning Goal: Distinguish between association and causation. Identify lurking variables that may explain an observed relationship.

Estimated time: 70 minutes

Big Ideas:

- Correlation is not the same as causation. (Correlation is the first step in looking for a cause-and-effect relationship, but it is only the first step.)
- Lurking variables may explain the correlation between explanatory and response variables.

Suggested set up (about 25 minutes)

Our suggested set-up is a short lecture with an interesting example. Here are our suggested lecture notes:

At this point we know that statisticians developed a way to measure the variability in a scatterplot using the correlation coefficient. But a strong correlation does not mean that the explanatory variable is actually causing a change in the response variable. In other words, a common mistake that people make is to confuse *correlation* and *causation*.

Why is this confusion such a big deal?

Well, this confusion is a particularly important because we often look for cause-and-effect relationships to guide our decision-making. For example, it is now commonly accepted that cigarette smoking causes lung cancer. There is now strong enough evidence from many studies, that the correlation between smoking and cancer is understood to be a cause-and-effect relationship, so doctors advise against smoking. But many studies only provide evidence of a correlation, and we should be cautious about interpreting this correlation as proof of a causal link.

Now project the following excerpt from an article in the Huffington Post.

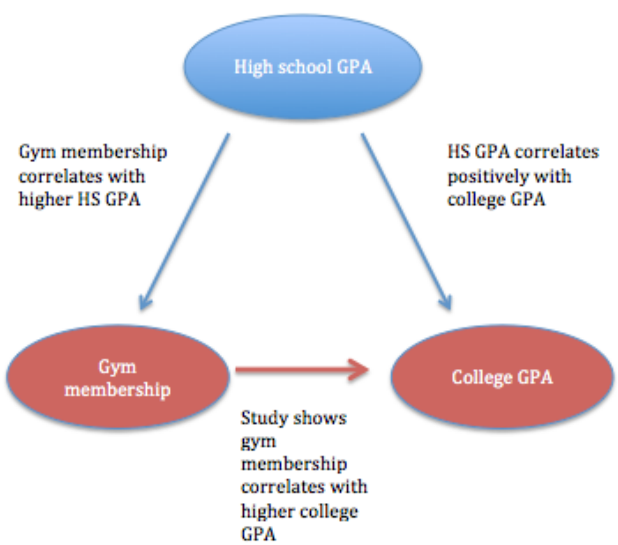
If You Want A Higher GPA, Study Shows You Should Join A Gym
The Huffington Post | By Nina Friend
Posted: 07/11/2014 2:57 pm EDT Updated: 07/11/2014 2:59 pm EDT
Can pumping iron pump your grade point average? New research from Michigan State University says yes.

An MSU study released this week compares 4,843 freshman and sophomores on the basis of whether they had membership to the school's recreational sports and fitness centers. The findings are significant: the students who belonged to the gym had higher GPAs than those who didn't. …

Lecture notes continued:

In this study, the researchers also found that students who belonged to a gym when in college also had higher high school GPAs. Obviously, students with higher HS GPAs will tend to have higher college GPAs. So HS GPA could be explaining the correlation between gym membership and college GPA. HS GPA is an example of a lurking variable that may be responsible for the correlation between gym membership and college GPA.

Project this diagram to show how a third variable (the lurking variable) is interfering with our ability to interpret the correlation as a cause-and-effect relationship.



Lecture notes continued: Finding a correlation between two variables is the first step in establishing a cause-and-effect relationship between the two variables, but it is just a first step. Later in the course we will discuss how researchers and scientists design studies with the goal of providing stronger evidence for a cause-and-effect relationship. But for now we will just practice being skeptical that a correlation is evidence of a causal link. We will look at a correlation between two variables and identify potential lurking variables that could be explaining the relationship we see.

Suggested implementation (about 30 minutes)

Either use iterative cycles of Think-Pair-Share to work through (1) and (2) or do group work followed by an Ambassador Exchange.

Suggested closure (about 15 minutes)

Think-Pair-Share on the question: “What is the main point of this activity?”

After all this cautionary discussion, students may be wondering why we do correlation at all. Emphasize that correlation is a powerful tool for exploring linear *relationships.* It is the first step in establishing a cause-and-effect link between two variables.

9.1 Introduction to Linear Regression

Learning Goal: For a linear relationship, use the least squares regression line to summarize the overall pattern and to make predictions.

Estimated time: 65 minutes

Big Ideas:

- If the relationship has a linear form, we can summarize this pattern with a line (duh!). This is analogous to summarizing a dotplot with a measure of center.
- This line can be used to make predictions for specified values of the explanatory variable.
- The predictions are probably more accurate if there is not a lot of scatter (variability) about the line. This is analogous to saying that a mean or median is a more accurate summary for distributions without much variability.

Suggested set up (30 minutes)

We suggest that you do #1 and #2 together as a class.

- Read the introduction out loud to focus the class or call on a student to read out loud.

<u>Discussion of (1)</u>

- Read the introductory material in #1 and ask a student to paraphrase the task (e.g. to see if one of the three students in the table could be the mystery student).

- (1a): Give students a minute to make a prediction about the mystery student's height based on a forearm length of 10 inches. Then call on students to share their predictions and methods. (The goal here is to make sure students are relating their prediction to the pattern in the data.

  If they are struggling here, suggest some ridiculous predictions (e.g. 56 inches or 74 inches) and ask students to articulate why these predictions are not good. Then ask them to provide predictions that are more reasonable. Reasonable height predictions fall between 62 and 68 inches, with 65 inches a good "average" prediction.)

- (1b): Give students "think time" in pairs, then call on students to share their reasoning. If students are struggling, suggest that they plot the three Jane Does assuming a 10-inch forearm. Based on this data, it is easy to see that Jane Doe #3 is probably too tall for this forearm length.

<u>Discussion of (2)</u>
- Introduce this problem as a method for coming to agreement on a "best" predicted height for a student with a 10-inch forearm (so that we will all have the same prediction, instead of multiple predictions as in #1).

- Again someone can read the introductory material out loud.
- Think (1 min)–Pair (1 min)–Share (2 min) for each part (a)-(c).
  [Answer for (2c): Jane Doe #1 is our best guess at who is the mystery student because her height is the closest to our prediction for a 10-inch forearm.]

Suggested Implementation (25 mins):

Let students work in randomly assigned groups on the rest of the activity.

If groups finish early, shuffle groups that finish early so that students can compare answers with people who were not in their group. Then assign students to prepare a presentation of #5 and assign others to serve as Roving Tutors.

Areas of potential difficulty:

(3a): Some students don't know how to convert to inches from feet and inches.

(3c): The answer to 3c can go either way. Both heights (67 and 68 inches) are on the upper edge of the data and neither is the predicted height. Be flexible in terms of the answer but make sure that students explain their argument for their answer. Students could argue that the model predicts that the bone belonged to a shorter person. Alternatively, they could argue that both heights are reasonable because there are 3 women in the data set with forearm lengths close to 9.6inches and heights similar to Amelia. Both are good reasoning.

#4 and #5: Many students, when asked to choose which prediction is more accurate, start to compare the value produced by the equation with the answer they get if they eyeball it on the graph, instead of looking at the variability in the data. As noted earlier, they do not understand that the graph is a visual representation of the equation and that if they measure it accurately, the "graph" value and the equation value are the same thing. They may choose the more "accurate" graph to be the one where they can more accurately estimate the graph value, instead of looking at how the data is spread about the line. Once they understand that the issue is which set of data has a stronger correlation, they breeze through the rest of the activity.

Suggested Closure (10 min):

Discuss #5, letting students demonstrate with a document camera how they made their predictions. Elicit both methods: equation and graph.

In the discussion of 5b, if someone does not bring in the concept of "strength" or correlation coefficient, ask how the answer to 5b is related to these ideas.

9.2 Introduction to Linear Regression

Learning Goal: For a linear relationship, use the least squares regression line to summarize the overall pattern and to make predictions.

Learning Objective: Interpret the rate of change (slope) and initial value (y-intercept) for regression lines.

Estimated time: 50 minutes

Suggested set up (15 min)

Work through the example at the beginning of the activity with the class. If you have time, you might want to do another example with a scatterplot that has a positive association and a different context.

For example, use StatCrunch and the Cereal data to find the regression equation for Protein vs. Consumer Report rating.

Simple linear regression results:
Dependent Variable: Rating
Independent Variable: Protein
Rating = 27.290277 + 6.0798403 Protein
Sample size: 77
R (correlation coefficient) = 0.47637071

Our experience is that students have difficulty interpreting parameters. Therefore, many instructors use a very formulaic approach to teaching this and resort to providing template sentences for students to fill in.

Y-intercept: When the (x variable) is zero (units), the (y-variable) is (y-intercept) (units).

e.g. When protein is 0 grams per serving, the rating is 27 points. In other words, when there is no protein in a cereal, the predicted cereal Consumer Rating is 27.

Rate of Change: For every extra (one x-variable unit), the (y-variable) goes up/down by (rate of change, y variable units)

e.g. For every extra gram per serving of protein, the Consumer Report rating goes up by about 6 points. In other words, the Consumer Report rating changes at a rate of 6 points per gram of protein.

If you use a formulaic approach to interpreting parameters, students should be allowed the flexibility to write interpretations that make sense mathematically and fit the context of the problem.

Suggested implementation (25 min)

Students can be confused by the instructions, “interpret the y-intercept and slope,” so before groups start working, make sure that they understand that the goal is to write a sentence explaining what the y-intercept means and a sentence explaining what the slope means. Both sentences will reference the explanatory and response variables and include units for these variables. There are no calculations in this activity.

Areas of potential difficulty:

- Some of the interpretations are nonsensical (#3 and #4 have nonsensical y-intercepts). You can treat this in the context of the y intercept being significantly outside the normal variable's value; in other words, inappropriately extrapolating the data. Or the sentences can be just considered straight meaningless. While it is OK to use the template sentence in the case of even a meaningless statement, encourage students to follow that with a comment about it being meaningless and why.
- #6: interpreting the y-intercept and slope can be difficult here because the units are in thousands. When students are working in groups, double check that they understand that 5.1 thousand is the same as 5.1 x 1,000 = 5,100 and 0.2 thousand per year is the same as 200 per year.

Closure (10 min):

Open a data set in StatCrunch and find a regression equation.

Think-Pair-Share to have students to interpret slope and y-intercept. Repeat with a different data set.

Assessing the Fit of a Line (15 minutes)

NOTE: Canvas has a page (Linear Regression (4 of 4)) and a quiz on assessing the fit of a line using r^2 and standard error of the regression. At the core of statistics is “model + error,” so these are important concepts.

Consider spending 15 minutes or so discussing r^2 and S_e.

Big ideas:

- **R-sq** gives us information about whether we have the right explanatory variable in our linear regression model. A large R-sq value tells us that the explanatory variable explains a large proportion of the changes in Y; our linear model based on X is good at explaining the variability in Y.

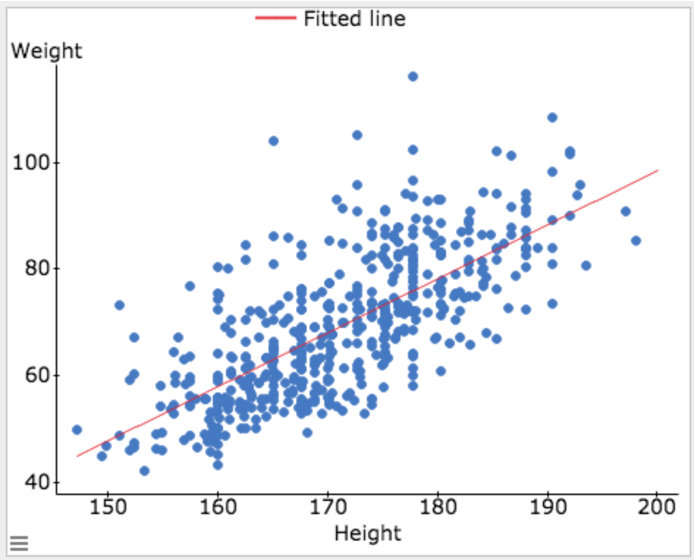
- **Standard error** is the standard deviation of the prediction errors. It is roughly a measure of the average amount that the predicted Y-values deviate from the observed Y-values, which is an estimate of the average distance that the data points are from the line. In other words, S_e is roughly the average amount of prediction error for the regression line.

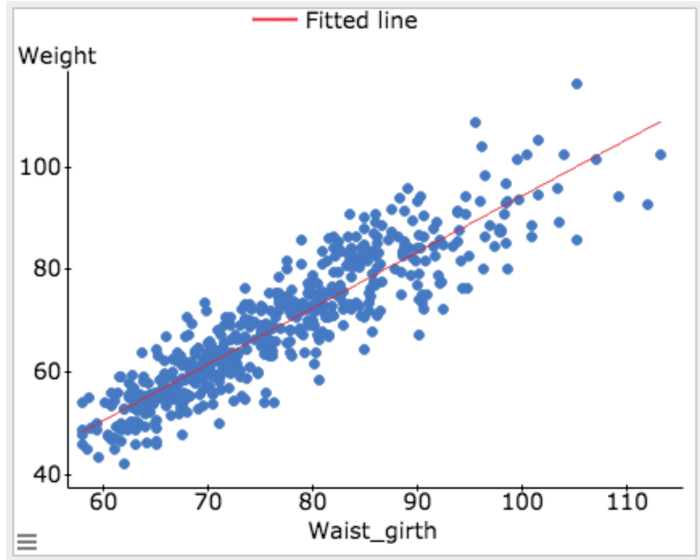
Example:

We suggest that you use the Body Measurements data set and investigate the question: which variable is a better predictor of Weight: Height or Waist Girth?

Use StatCrunch to find the linear regression equation. The output will contain r, r^2, and S_e. Height vs. Weight has a smaller r^2 and a larger S_e, so Height is the weaker predictor of Weight.

We provided StatCrunch outputs below that you can use with the document camera.





Simple linear regression results: Dependent Variable: Weight Independent Variable: Height Weight = -105.0 + 1.0Height Sample size: 507 R (correlation coefficient) = 0.72 R-sq = 0.52 Estimate of error standard deviation: 9.31	Simple linear regression results: Dependent Variable: Weight Independent Variable: Waist_girth Weight = -15.2 + 1.1 Waist_girth Sample size: 507 R (correlation coefficient) = 0.90 R-sq = 0.82 Estimate of error standard deviation: 5.7

9.4 Unit 3 Group Project

Run the Unit 3 Project like a "conference style" poster session. This project requires the use of StatCrunch.

Estimated time: about 70 minutes

Below are general notes for facilitating "Conference style" Poster Presentations.

Overview of how to run a "conference style" poster presentations

- Students work in groups on a difficult problem.
- Each group makes a poster to summarize their analysis and conclusion.
- One group member stays at the poster to present the findings.
- The rest of the group rotates from poster-to-poster to hear short presentations, similar to a gallery walk for a poster session at a conference.
- Students who are not presenting have a listening/responding protocol that fosters active engagement during each presentation.

What materials are needed?

- Access to StatCrunch
- Access to the data through our StatCrunch group (Math 110 Los Medanos College)
- Graph paper and plain notebook paper (2-3 sheets per group)
- Poster paper (one to two sheets for each group of 4 students)
- Tape
- Markers (1-2 for each group of 4 students)
- Post-its (3x3in, two colors; for a class of 32, 168 post-its in each color)

Facilitation notes (with specific suggestions for this project.)

<u>Set-up:</u> Approximately 5 minutes.
Review the instructions. Explain how to access the data sets.

<u>Group work</u>: Approximately 30 minutes.

- Group work: Students work in groups of 4 to answer the question using observations from the data. Each group makes a poster with graphs, diagrams, tables, verbal descriptions, and/or calculations from their StatCrunch analysis.

- Poster construction: All students should be engaged in the making of the poster, e.g. some can translate StatCrunch graphs onto graph paper, others can write the verbal descriptions on notebook paper, etc. These parts can be taped to the poster.

- Poster display: Hang posters so that the four problems occur in a side-by-side sequence, followed by another sequence of 4 posters representing the 4 problems. This way a rotating student sees all four problems, including another group's perspective of the problem (s)he worked on.

Poster presentations: Approximately 20 minutes

- Dry run: 3-5 minutes for groups to practice prior to the gallery walk;
- Gallery walk: four 3-min rounds (rotating group gets to hear presentations on each of the 4 problems.)

Instructions to give to students:

- One student will be the presenter. (The instructor can choose the presenter using random assignment or choose someone who looks disengaged or lost.).
- Groups have 3-5 minutes to do a dry run. This gives the presenter time to practice and get feedback from the group.
- Groups will rotate together (minus the presenter) when the lights flicker (or alarm sounds). Each round will last about 3 minutes.
- Before rotations begin, give stacks of the two colors of post-its to each presenter.
- Each student listening to a presentation will provide feedback to each presenter on post-its. One color is for a kudo: a positive note that captures something that was clear or compelling in the analysis. Another color is for a piece of constructive feedback: a note that highlights something that was unclear or perhaps incorrect or something that could be expanded in the analysis. Each student must leave two post-its (one of each type) at each poster.

Note: With 4 rotations, each presenter will do his/her presentation 4 times. This allows the presenters the opportunity to improve their understanding of their group's analysis and to improve their explanation in a low-stakes environment. This is a reason to pick a student who seems disengaged or lost.

Note: Insist that each student leaves both a kudo and a piece of constructive feedback at each poster. This helps students learn to mentally engage with a presentation, to treat their peers' ideas with respect, and to learn to disagree in positive and constructive ways. Monitor this during the gallery walk by visually checking to see if there is an equal number of post-its in the two colors at each poster and that are each round, the number of post-its increases by 6 (3 of each color, left by each student.)

Potential areas of difficulty

While students are working in groups, circulate and check for the following:

- Correct identification of the explanatory variable with correct scatterplots
- Use of r, r^2 and/or standard error to identify the best predictor.
- Correct prediction

Closure: Approximately 15 minutes (or more if you have time.)

(5 mins) Review feedback: When the groups have rotated back to their own poster, they will naturally begin to look at the feedback on the post-its. They will usually also begin to explain what was on other posters to the presenter, who will probably be asking about this.

(10 mins) Discussion

- Ask students to return to their seats for discussion.

  Based on your observations and how much time you have, focus the discussion on the area of greatest need. Here are some ideas:

- Which variable gives the best prediction? Ask students to identify a poster that did a good job explaining WHY one variable was a better predictor. Briefly discuss the use of r, r^2 and/or standard error.

- Feedback: Ask a few of the presenters to share the most helpful piece of feedback that they received. Helpful feedback tends to be specific. Emphasize that feedback is an invaluable way to improve your work and that it is a fundamental part of working productively in teams.

Follow-up assignment: Consider assigning an individual write-up or discussion board item where each student answers the question of their choice in writing with support of the data.

The purpose of this assignment is increase accountability in group work, motivate students to learn from each other, and communicate that work on real life problems takes time and effort. We will use this cycle of brainstorming, presentation of preliminary ideas, feedback, revision throughout the many projects we do in this course.

10.1 Relationships in Categorical Data

Learning Goal: Use a two-way table to analyze the association between two categorical variables.

Estimated time: About 50 minutes

Big Ideas:

- When comparing two groups' categorical responses, comparing counts is problematic if the groups are not the same size. Use percentages instead, to scale the groups to the same size of 100.
- If there is "no association" between two categorical variables, then the explanatory variable does not affect the distribution of the response variable. In other words, when there is no association, the distribution of the response variable will closely resemble the distribution of conditional percentages for each value of the explanatory variable.

Suggested Set-Up:
Have a student read the introduction to the activity.

Suggested implementation (40 mins)

#1-#4 Think-Pair-Share, #5 in groups.

- #1-3 (15 minutes)
  Work as a class through #1-3 using quick iterative cycles of Think-Pair-Share.
  - During the discussion of (1), ask students to think of variables that have numerical values but are categorical. Some examples are social security numbers, student identification numbers, and area codes are all good examples.
  - During the discussion of (3), ask students to identify the variables, categorize them as quantitative or categorical, and identify which is the explanatory and which is the response variable. (Why is this important? It helps us find conditional percentages appropriate to determining if there is an association between the two variables.)

- #4 (15 mins)
  Start with a few quick comprehension questions to make sure students understand the table, then do Think-Pair-Share.

  - (2 mins) Call on students at random to answer questions such as, how many students took the survey? How many female students took the survey? How many students sit in the front of class? How many female students sit in the front of class? What does the number 47 tell us? The number 25?
  - (5 mins) Think-Pair-Share with everyone working #4a and #4b.
  - (8 mins) To move more quickly through the rest of #4, divvy up the work on c, d, and e with different segments of the room assigned to Think-Pair-Share on different parts. Discuss parts f and g together.

- #5: Group work (10 mins)

  Randomly assign students to groups to work on #5. Circulate to check their progress and intercede if necessary.

Areas of potential difficulty:

- Students may ask, "how much do the two percentages have to differ before we say that there is an association?"

  This is a great question that is best answered when students learn inference procedures that determine if a difference is statistically significant. For now, tell students that this is a judgment call. We expect some variability in samples even when there is no association between the two variables, but a lot of variability suggests that one variable may be associated with the other. We will not define how much of a difference is "a lot"; again, this is a judgment call at this point.

- If students struggle with constructing these ratios,
  - Emphasize that the comparison group totals are the denominators. For example, if we are comparing males to females, then we need the counts of males and females.
  - The numerators use the other variable to describe subgroups. For example, when comparing males and females, we might count back sitters in each gender group.

- In the discussion students may struggle with writing accurate sentences.
  If they are struggling, consider using a template such as, "Of the 150 females, 14.67% (22 out of 150) sit at the back." In our experience leading with "of the" comes easier to students than leading with the percentage, e.g. "14.67% of the females sit at the back" seems to lead to more errors later.

Suggested closure (10 minutes)

If there is time you can assess students understanding by asking them to investigate a new question, "Are movies produced by the Big6 studios more likely to win an Oscar?"

- Project the two-way table given below.
- After 2 minutes of think time, do several 2-minute speed dates, then call on random students to share their analysis. Ask follow-up questions of other students to check for understanding.

	Oscar Winner NO	Oscar Winner YES	Total
Big6	20	19	39
Other	21	12	33
Total	41	31	72

This data comes from IMDb.com and describes the 72 movies with the top U.S. box office sales of all time.

10.2 Introduction to Probability and Risk

Learning Goal: Use a two-way table to estimate probability and risk.

Big Ideas:

- Risk as the probability of a negative outcome;
- Comparing risks using conditional probabilities;
- Comparing conditional risks using percentage point difference, risk ratios, and percent change.

Estimated time: about 50 minutes

The calculations here are the same calculations that we did in 10.1 with conditional percentages. What is new is the use of probability language and the different ways to compare conditional probabilities to quantify the difference.

Suggested Set-up (about 5 minutes)

Have a student read the introduction out loud.

Suggested implementation (About 20 minutes)

<u>Class discussion #1</u> (about 5 mins)

Walk through #1 as a class.

For #1d, highlight the use of the phrase “percentage point difference” and explain why we do not say “percentage difference” when subtracting. This is due to the fact that the denominators differ; we are not calculating percentages using the same base.

You might also ask students to discuss which comparison is easiest to understand and which is the most informative when making a comparison. This is an opportunity to discuss percentage point difference vs. relative difference, which is a difficult concept for some students.

Percentage point difference is easy to understand but it can be misleading. For example, a 5-percentage-point difference is huge when comparing 6% to 11%, but it is less impressive when comparing 94% and 99%. Relative difference is more helpful. When comparing 6% to 11%, the relative difference is almost 2 (1.8 to be exact); one percentage is almost twice as big as the other. When comparing 94% to 99%, the relative difference is 1.05. A relative difference of 1 says the numbers are equal, so a relative difference of 1.05 is telling us that the numbers are almost equal.

<u>Group work #2-3</u> (about 15 mins)

Randomly assign groups of 4 to work on #2 and #3. Monitor groups closely on calculations and interpretations. Step in to guide if you need to.

Potential areas of difficulty:

- Students may have difficulty setting up appropriate conditional percentages.
  - Using the idea of explanatory and response variable may help. The explanatory variable determines our comparison groups and dictates which counts go into the denominators.
  - Note that some two-way tables have the explanatory variable as the row variable, in which case, the row totals will be the denominators of the conditional percentages. Some have the explanatory variable as the column variable, in which case, column totals will be used in the denominators.

- Students may struggle with interpreting their comparisons. Keep an eye out for the following issues and work with students in their groups on correcting inaccurate statements.
  - When subtracting two percentages, statisticians generally interpret the difference in percentage <u>points</u>, instead of a difference in percentages. This is a subtle distinction but important. The phrase *percentage <u>point</u> difference* highlights that we are subtracting percentages that may come from different sized groups (different bases for the percentage).
  - When dividing percentages, students may get confused about how to set up the ratio and how to interpret it. In general, it is easiest to set up the ratio with the smaller number in the denominator because the result is greater than one, which is easier to interpret and more like what we hear in everyday conversation.
  - When calculating percent change, students may have difficulty determining what to use as the base of the calculation (in the denominator). In general practice, the placebo (no treatment) or status quo is used as the point of comparison in the base.

Suggestions for Closure (About 20 minutes)

<u>Ambassador Exchange</u>:
  - Ambassadors visit host groups (about 5 minutes) and return to home group for discussion (5 minutes.)
  - As a class discuss any issues not resolved at this point (5 minutes).

OR

<u>Class discussion</u>: Have students present solutions using the document camera.

OR

<u>Assessment with a new problem</u>: Have students use StatCrunch to select a random sample of StatCrunchU students and determine the probability that the student is female and whether it is more likely to select a female from the freshmen class or from the senior class. (Log into StatCrunch, go to **Resources**, scroll down to **Take a sample from StatCrunchU,** click on **StatCrunchU.** Do not complete the survey. Instead scroll down to **Sample size** and choose n. Then click **Survey**.)

10.3 Unit 4 Summary Lab

Learning goal: Use a two-way table to analyze the association between two variables.

Specific Learning objectives:
- Given a two-way table, calculate and interpret risk probabilities.
- Compare risk by calculating the percentage point difference, the relative risk quotient and the percent increase or decrease in risk.

Facilitation suggestions (about 40 mins)

- (20 mins) Do the lab in groups.
- Have groups that finish early, compare answers. Assign them to present a problem. Look for clear documentation of work and clear sentences communicating meaning of numbers. Others can serve as Roving Tutors.
- (20 mins) Follow with student presentations using the document camera. After each presentation, ask comprehension questions, randomly calling on students in the "audience" to answer.

Record an effort grade for the lab.

10.4 Unit 4 Group Project

Run the Unit 4 Project like a “conference style” poster session. This project requires the use of StatCrunch.

Estimated time: about 60 minutes

Below are general notes for facilitating “Conference style” Poster Presentations.

Overview of how to run a “conference style” poster presentations

- Students work in groups on a difficult problem.
- Each group makes a poster to summarize their analysis and conclusion.
- One group member stays at the poster to present the findings.
- The rest of the group rotates from poster-to-poster to hear short presentations, similar to a gallery walk for a poster session at a conference.
- Students who are not presenting have a listening/responding protocol that fosters active engagement during each presentation.

What materials are needed?

- Access to StatCrunch
- Access to the data through our StatCrunch group (Math 110 Los Medanos College)
- Graph paper and plain notebook paper (2-3 sheets per group)
- Poster paper (one to two sheets for each group of 4 students)
- Tape
- Markers (1-2 for each group of 4 students)
- Post-its (3x3in, two colors; for a class of 32, 168 post-its in each color)

Facilitation notes (with specific suggestions for this project)

<u>Set-up:</u> Approximately 5 minutes.
Review the instructions. Explain how to access the data sets.

Note: Option 5 question #1 is unclear. To force the use of conditional percentages, make the following change to that question: **Compared to other movie studios**, are the Big 6 **studios** more likely to choose a person of color as a lead star?

<u>Group work</u>: Approximately 20 minutes.

> Group work: *The instructions say “choose one of the five options,” but this will probably take too much time. Instead, assign problems to groups*. Students work in groups of 4 to answer the question using observations from the data. Each group makes a poster with graphs, diagrams, tables, verbal descriptions, and/or calculations from their StatCrunch analysis.

Poster construction: All students should be engaged in the making of the poster, e.g. some can translate StatCrunch graphs onto graph paper, others can write the verbal descriptions on notebook paper, etc. These parts can be taped to the poster.

Poster display: Hang posters so that the five problems occur in a side-by-side sequence, followed by another sequence of 5 posters representing the 5 problems.

<u>Poster presentations</u>: Approximately 20 minutes
Dry run: 3-5 minutes for groups to practice prior to the gallery walk;
Gallery walk*: five 3-min rounds* (rotating group gets to hear presentations on each of the 5 problems.)

Instructions to give to students (if necessary):

- One student will be the presenter. (The instructor can choose the presenter using random assignment or choose someone who looks disengaged or lost.).
- Groups have 3-5 minutes to do a dry run. This gives the presenter time to practice and get feedback from the group.
- Groups will rotate together (minus the presenter) when the lights flicker (or alarm sounds or whatever you are using as a signal.) Each round will last about 3 minutes.
- Before rotations begin, give stacks of the two colors of post-its to each presenter.
- Each student listening to a presentation will provide feedback to each presenter on post-its. One color is for a kudo: a positive note that captures something that was clear or compelling in the analysis. Another color is for a piece of constructive feedback: a note that highlights something that was unclear or perhaps incorrect or something that could be expanded in the analysis. Each student must leave two post-its (one of each type) at each poster.

Note: With 5 rotations, each presenter will do his/her presentation 5 times. This allows the presenters the opportunity to improve their understanding of their group's analysis and to improve their explanation in a low-stakes environment. This is a reason to pick a student who seems disengaged or lost.

Note: Insist that each student leaves both a kudo and a piece of constructive feedback at each poster. Monitor this during the gallery walk by visually checking to see if there is an equal number of post-its in the two colors at each poster and that are each round, the number of post-its increases by 6 (3 of each color, left by each student.)

Potential areas of difficulty

As you circulate, help groups with the following if there are problems:

- Calculation of conditional percentages

- Making pie charts or bar charts. We have not practiced this at all, but at this point students may be able to figure it out. For pie charts, there should be a pie for each value of the explanatory variable, with areas representing the distribution of the response variable. For bar charts, the explanatory variable is on the horizontal axis.

Closure: Approximately 10 minutes (or more if you have time.)

(5 mins) Review feedback: When the groups have rotated back to their own poster, they will naturally begin to look at the feedback on the post-its. They will usually also begin to explain what was on other posters to the presenter, who will probably be asking about this.

(5 mins) With groups still at their posters, choose one of the following for discussion (depending on your observations about student need):

- Conditional percentages: Every group used conditional percentages. Why is that? [Answer: each research question involved two categorical variables.]
  Why is it important to identify which variable is the explanatory variable?
  [Answer: the explanatory variable defines the comparison groups. The denominators of the conditional percentages come from the explanatory variable.]

- Pie charts or barcharts: Discuss how the explanatory variable and response variable are represented in each type of graph by highlighting this in some of the posters. As noted above, for pie charts, there should be a pie for each value of the explanatory variable, with areas representing the distribution of the response variable. For bar charts, the explanatory variable is on the horizontal axis.

- Feedback: Ask a few of the presenters to share the most helpful piece of feedback that they received. Helpful feedback tends to be specific. Emphasize that feedback is an invaluable way to improve your work and that it is a fundamental part of working productively in teams.

Follow-up assignment: Consider assigning an individual write-up or discussion board item where each student answers the question of their choice in writing with support of the data.

The purpose of this assignment is increase accountability in group work, motivate students to learn from each other, and communicate that work on real life problems takes time and effort. We will use this cycle of brainstorming, presentation of preliminary ideas, feedback, revision throughout the many projects we do in this course.

Discussion of "Why You Should Be Proud of Making Mistakes"

https://www.lifehack.org/299971/this-why-you-should-proud-making-mistakes

This is an optional class activity. Use it if you think your class will benefit. You might have this discussion of the benefits of making mistakes either before or after the Teach a Problem exam preparation activity.

Learning goal: See mistakes as an important conduit to learning.

Facilitation suggestions (about 30 minutes)

Distribute the blog "Why You Should Be Proud of Making Mistakes." A printable copy follows these notes.

Give students about 10 minutes to read the blog.

Share a mistake that you have made that you can be proud of. Highlight the reason in the list of 10 that underscores why this mistake helped you learn something.

Ask students to do the same. Share a mistake that they have made that they can be proud of, and highlight the reason in the list of 10.

This Is Why You Should Be Proud of Making Mistakes

Crystie Lim blog https://www.lifehack.org/299971/this-why-you-should-proud-making-mistakes

We all know too many people who are afraid of making mistakes because doing so makes them feel horrible about themselves. But the truth is, mistakes can be like gifts. Any successful person can tell you that without the mistakes they made on their journey, they wouldn't be where they are today. Here are 10 reasons why you should be proud of making mistakes.

1. Mistakes help us gain knowledge.
We can gain so much knowledge from our mistakes, and all it takes is the willingness to learn from them. We get to know what works and what doesn't from each error we come across. Without mistakes, we lose countless opportunities to gain valuable knowledge and learn lessons.

"The only real mistake is the one from which we learn nothing."—John Powell

2. Mistakes trigger our creativity.
When what we are attempting is not working, we look for a new solution, which enables us to think outside the box. Without mistakes, we lose chances of experiencing the creative side of ourselves.

"The man who makes no mistakes does not usually make anything."—William Connor Magee

3. Mistakes help us learn to be resilient.
We get to understand that adversity is needed to overcome challenges in life. We know then that being flexible is important, as we can't move forward if we stay in the same place. Without making mistakes, we would stay in our comfort zone, which leads us to inflexibility when things don't work the way we want them to.

"The more we can embrace failure, the more we will be able to open to it and the more confident and resilient we will become."—Karen Kimsey-House

4. Mistakes teach us about humanity.
Mistakes teach us the importance of staying humble. We learn that we are only human and that mistakes are inevitable. We also learn that humility is what make us beautiful.

"If you have made mistakes, even serious ones, there is always another chance for you. What we call failure is not the falling down, but the staying down."—Mary Pickford

5. Mistakes provide us references.
We obtain references each time we try different approaches as we attempt to achieve our goals. We become better each time we try something new. We wouldn't gain those references if we didn't make mistakes.

"I have not failed. I've just found 10,000 ways that won't work"—Thomas Edison

6. Mistakes help us obtain new ideas.
Making mistakes is one of the most effective ways to gain new ideas. Our mistakes force us to push limits and find new things that inspire us.

"The only sure way to avoid making mistakes is to have no new ideas"—Albert Einstein

7. Mistakes give us courage.
We become a stronger person each time we acknowledge our mistakes and move on to do better next time. We come to understand that being brave is all about admitting to our faults and growing from them.

"Mistakes are always forgivable, if one has the courage to admit them."—Bruce Lee

8. Mistakes make us wiser.
We learn to appreciate those who have helped us, and learn to deal with the people who have done us wrong. We learn to be wiser each time we make a mistake, because we get to see a different side of the people we thought we knew. Without making mistakes, we would not get to see the different sides of those we deal with.

"Some people come into your life as blessings. Some come into your life as lessons."—Mother Teresa

9. Mistakes teach us how to experiment.
We are experimenting each time we try a new approach to creating something. We often create amazing things when we innovate, be it a product or something personal. But innovation comes from experimenting, which often leads to making mistakes. Without those mistakes, we wouldn't be able to innovate, and create better things. We would also not be able to become a better person without knowing what we did wrong.

"Sometimes when you innovate, you make mistakes, It is best to admit them quickly, and get on with improving your other innovations."—Steve Jobs

10. Mistakes help us to better understand ourselves.
Mistakes helps us understand that our biggest enemy can be ourselves. We learn to reflect on our own mistakes. By overcoming our shortcomings, we become stronger people as we come to know our own strengths and weaknesses. It wouldn't be as easy for us to understand ourselves without the mistakes we have made.

"In the business world, the rearview mirror is always clearer than the windshield."—Warren Buffett

11.1 Introduction to Probability

Learning Goal: Interpret probability as a long run relative frequency.

Estimated time: 30 minutes

We have only allotted 30 minutes for this activity; therefore, we recommend an interactive lecture (with judicious use of Think-Pair-Share) instead of group work. Here are our suggestions for how to interactively work through 11.1:

1) Begin with a quick summary of the introduction. You might ask students to give an example of an event with probability 1 and an event with probability 0. (4 minutes)

2) Discuss #1 to motivate the need for empirical probability estimates using relative frequencies. (6 minutes)

   - What is the probability that a toss of a fair coin lands on heads? (P(H)=0.5) How do we know? (two equally likely outcomes)
   - What is the probability that the toss of a bottle cap lands right-side up? (unknown: two outcomes but not equally likely; we need to toss a lot of bottle caps and see what happens)
   - What is the probability that a randomly selected LMC student is a female? (Two possible outcomes, but about 54% of the LMC student population is female according to 2018 student demographic information; this relative frequency is the probability of selecting a female at LMC in 2018.)

Big idea: When outcomes are equally likely, we can estimate the probability theoretically as one out of the possible outcomes. When outcomes are not equally likely, we can estimate the probability empirically using a relative frequency.

3) For #2, conduct a demonstration for tossing a biased coin. (15 mins)

Getting a head when tossing a biased coin is an example of an outcome whose probability cannot be determined in advance just by knowing there are two possible outcomes to the experiment. You need to actually toss the coin and count heads.

To simulate tossing a biased coin, use the online applet
https://lmcstatistics.shinyapps.io/biasedcoin
Note: The applet randomly chooses P(heads)=0.2, 0.3, 0.4, 0.6, 0.7, or 0.8.

Notes on leading the Biased Coin Demonstration:

- From the podium, open the applet and select 10 tosses, then click the Toss button. The applet shows the frequency of heads and tails, and a graph of relative frequencies (number of heads / number of tosses) as the number of tosses increases.
- Using some of the dots shown in the graph, explain how the relative frequency relates to count of heads.
- Have students guess P(H) for this coin.
- Select an increasing number of tosses. Finally, select 1000 tosses. After 1000 tosses the relative frequency history should settle down to where the class can determine the empirical probability of heads visually from the graph. Press the Reveal p button to see the actual probability of heads.
- If there is time and interest, refresh the browser to reset the applet and redo the demonstration for a new biased coin.

End with a Think-Pair-Share on #2

Big ideas:

- When we do not know a theoretical probability (like with a weighted coin or loaded die), we can estimate it using a relative frequency, aka empirical probability.
- A large number of tosses (or rolls of a die) will tend to give a better estimate than a small number of tosses.

4) For #3, collect class data and discuss #3 (5 mins)

Big ideas:

- Relative frequencies can be used to estimate a probability.
- We can only make probability statements when chance is involved. When chance is involved, an event (like randomly selecting a student) has an unpredictable outcome in the short run, but in the long run there is a pattern that allows us to predict the likelihood of the outcome.

11.2 Probability Distributions

Learning Goal: Use probability distributions to identify typical and unusual events and, when appropriate, to determine an expected value.

Estimated time: about 50 minutes

We have only allotted 50 minutes for this activity; therefore, we recommend an interactive whole class discussion with Think-Pair-Share instead of group work.

Think-Pair-Share #1-#6 (about 40 mins)

- Give students time to read a segment quietly or call on a student to read out loud to the class.
- Think: On *check your understanding* questions, give students a minute or two of quiet time to solve the problem(s).
- Pair: After a minute, call time and give them a minute or two to confer with a neighbor.
- Share: Call on students at random to share answers and explanations. Check for agreement. If necessary, reinforce student explanations with smaller questions that others can answer to clarify concepts.

#7 Expected Value (10-15 mins)

Demonstrate calculating expected value in #7. See Canvas Module 11 - Probability Distributions for a Quantitative Variable (2 of 3) and (3 of 3) for ideas on how to tie the formula for expected value to the calculation of a weighted mean.

Areas of potential difficulty:

#2: Explaining why the properties of probability distributions make sense may be hard for students. Thinking of probability as relative frequency is key to both explanations.

#4: Students may have trouble identifying the probability distribution of a fair die because neither distribution shows a perfect uniform distribution. If necessary, you can use this applet to examine the expected variability in outcomes for a fair die as the number of rolls increases. **https://www.geogebra.org/m/UsoH4eNl**.

12.1 Probability Distributions for Continuous Random Variables

Learning Goals:

- Use a probability distribution for a continuous quantitative variable to estimate probabilities and identify unusual events.
- Use the mathematical model of a normal curve to estimate probabilities when appropriate.

Total time estimate: about 60 minutes

Technology: Students will need access to the internet to use StatCrunch.

Overview: This activity is structured like the previous activity with short examples followed by comprehension checks, but it also includes a group work section.

We suggest you facilitate an interactive discussion of the new material using quick iterative cycles of Think-Pair-Share as you did in the previous activity.

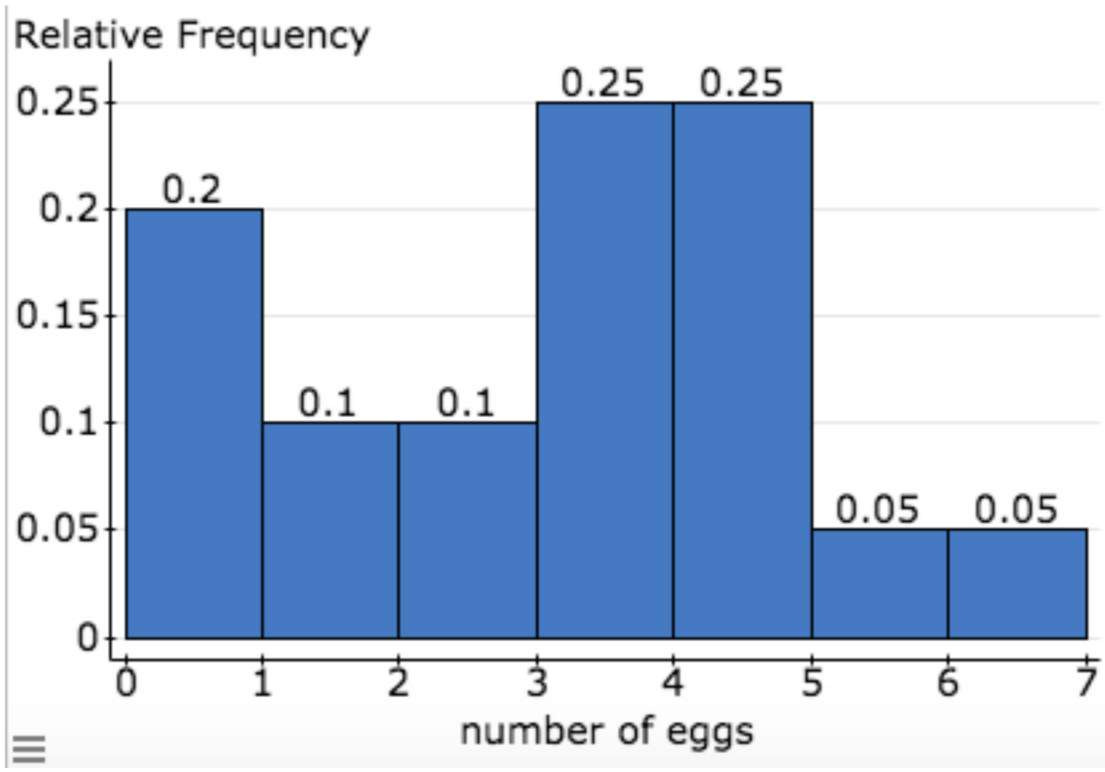
- Give students time to read a segment quietly or call on a student to read out loud.
- Think: On *check your understanding* questions, give students a minute of quiet time to solve the problem(s).
- Pair: After a minute, call time and give them a minute to confer with a neighbor.
- Share: Call on students to share answers and explanations. Check for agreement. If necessary, reinforce student explanations with smaller questions that others can answer to clarify concepts or correct mistakes.

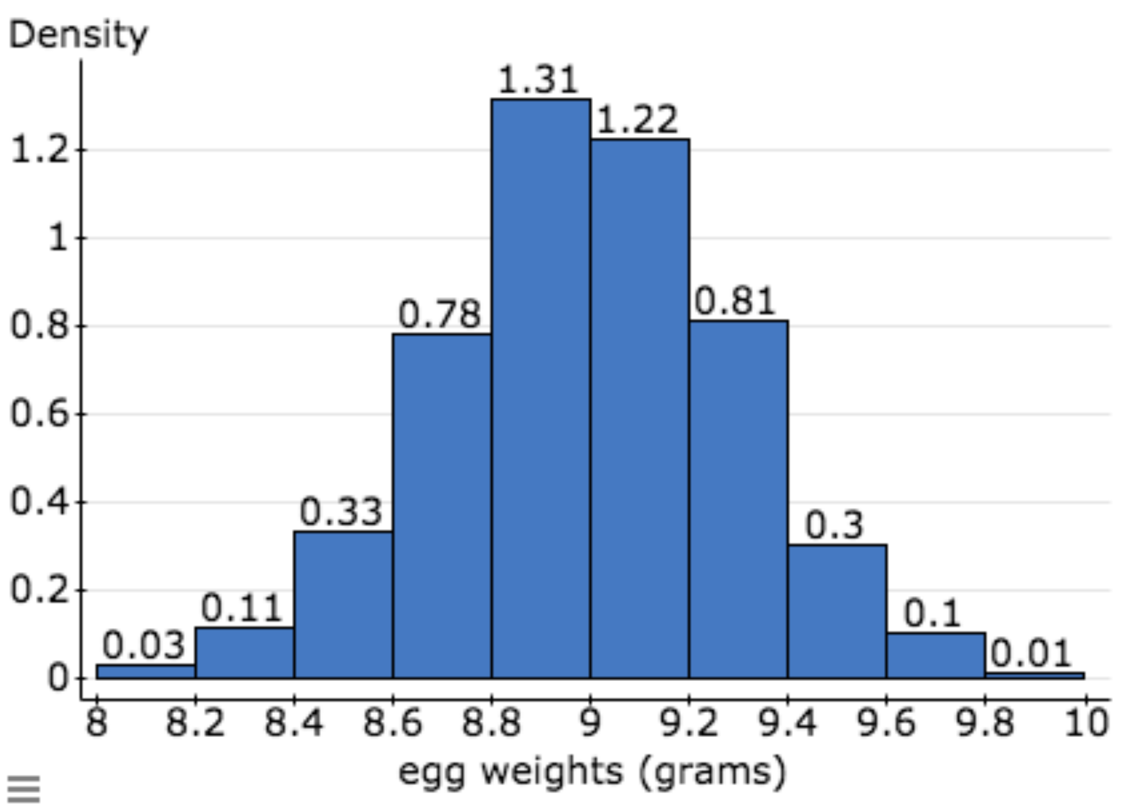
Suggested time allotments:

- Discrete vs. Continuous quantitative variables (5 minutes)
- Probability Distributions (13 minutes)
- Normal probability curve (12 minutes)
- Group work (15 minutes)
- Closure (15 minutes)

Potential areas of difficulty

Probability Distributions: The continuous probability distribution has density on the y-axis. Students may have difficulty understanding density and computing probability using a density histogram. With density, probability is AREA, not height. This foreshadows the use of area to estimate probabilities using a normal density curve. Do not worry if students do not fully understand density. Students do not need to understand the concept of density in order to understand that area is probability. **(**We provide larger versions of these two probability distributions on the next page in case you need them for class discussion.)


Relative Frequency
0.25
0.2
0.15
0.1
0.05
0
0.2
0.1
0.1
0.25
0.25
0.05
0.05
0
1
2
3
4
5
6
7
number of eggs


Density
1.2
1
0.8
0.6
0.4
0.2
0
0.03
0.11
0.33
0.78
1.31
1.22
0.81
0.3
0.1
0.01
8
8.2
8.4
8.6
8.8
9
9.2
9.4
9.6
9.8
10
egg weights (grams)

Normal curve

Students may struggle with the idea that the normal curve is a mathematical model. One way to drive this point home is to show the equation. Reassure students that we will not use this equation, but notice that the curve of the normal model depends on μ (the mean of the probability distribution) and σ (the standard deviation of the probability distribution.)

$$y = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{-(X-\mu)^2}{2\sigma^2}}$$

StatCrunch

Students will use StatCrunch in the group work.

Instructions for locating the StatCrunch Normal Calculator:

- Log into StatCrunch
- From the menu at the top, choose Open StatCrunch
- Under Stat, choose Calculators, Normal
- Enter the mean and SD and the X-interval.

Suggested Closure (15 minutes)

Here are some ideas for closure. Choose one.

1) Use speed-dating or Ambassador exchange so that students can check their answers on group work problems.
2) Student presentations: Let students present group work problems using the document camera and/or StatCrunch at the teacher's podium.
3) Have students individually work another problem using StatCrunch and compare answers with a neighbor. Share out and check that everyone got the same answer.

Extra problem: Birth weights have a normal probability distribution.

A study published in the *American Journal of Epidemiology* (Volume 141, Issue 12, 15 June 1995, Pages 1177–1187) compared birth weights of Caucasian, immigrant Chinese, and native Chinese infants. Means (standard deviations) in grams were 3,369 (567), 3,195 (493), and 3,171 (428), respectively.

What is the probability that an infant from each group weighs less than 2,500 grams (the definition of low birth weight in the U.S.)?
Answers: Caucasian 0.063, immigrant Chinese 0.079, native Chinese 0.058

Why does it make sense that native Chinese are the least likely to have a low weight birth when the mean birth weight for this group is the smallest?

12.2 Introduction to the Normal Distribution

Learning Goal: Use the properties of the normal density curve to estimate probabilities using the Empirical Rule and standardized scores.

Estimated time: about 60 minutes

This activity is structured like the previous activity with short examples followed by comprehension checks and concludes with a short group work section. StatCrunch is not required for this activity.

Facilitate #1-4 interactively with quick iterative cycles of Think-Pair-Share, calling on students at random to answer each part after they have had quiet time to think and a minute to confer with a neighbor. Check for agreement and, if necessary, ask smaller scaffolding questions to clarify concepts and let students correct mistakes.

Suggested time allotments:

General properties (10 minutes)
Empirical rule (15 minutes)
Group work (20 minutes)

Suggested Closure (15 mins)
Here are some ideas for closure. You will only have time for one.

Tag team problem:

- Give another Empirical Rule problem with a different context.
- Give everyone about 2 minutes to work the problem.
- Tag team: Randomly call on a series of students to tag team work at the board. Each student gets one minute to work toward the solution. Each successive student can erase or add to the previous student's work. Repeat with another problem if there is time.

Extra problem: The lifetime of Brite lightbulbs have a normal distribution with a mean of 400 hours and a standard deviation of 75 hours. What is the probability that a Brite lightbulb lasts less than 325 hours? Less than 250 hours?

Student presentation: Randomly choose students to present group work solutions. Use the document camera .

12.3 Standardizing Scores and the Standard Normal Probability Distribution

Learning Goal: Calculate and interpret a standardized score and use the Standard Normal probability density curve to estimate probabilities.

Estimated time: about 45 minutes

Technology: Students will need access to the internet to use StatCrunch.

This activity is structured like the previous activity with short examples followed by comprehension checks and concludes with a short group work section.

Facilitate #1-4 interactively as you have done before with quick iterative cycles of Think-Pair-Share in each segment.

Suggested time allotments:

Standardized scores (10 minutes)
Standard Normal (15 minutes)
Group work (10 minutes)

Suggested Closure (10 mins)
Here are some ideas for closure. You will only have time for one.

Tag team problem:

- Give another problem with a different context.
- Give everyone about 2 minutes to work the problem.
- Tag team: Randomly call on a series of students to tag team work at the board. Each student gets one minute to work toward the solution. Each successive student can erase or add to the previous student's work. Repeat with another problem if there is time.

Extra problem: The lifetime of Brite lightbulbs has a normal distribution with a mean of 400 hours and a standard deviation of 75 hours. What is the probability that a Brite lightbulb lasts less than 300 hours?

See if students can work the problem using both the Standard Normal Curve and the Normal Curve with mean 400, SD 75.

Student presentation: Randomly choose students to present solutions to #4. Use the document camera. For (4d) elicit solutions that use both the Standard Normal Curve and the Normal Curve with mean 22.8, SD 2.15.

12.4 Unit 5 Lab

Learning goal: Use the Empirical Rule, the Standard Normal and/or StatCrunch Normal Calculator to estimate probabilities using the Normal Curve.

This lab provides extra practice with the Empirical Rule, the Standard Normal and/or StatCrunch Normal Calculator to estimate probabilities using the Normal Curve. Most of the problems have multiple solution strategies using these ideas. For this reason, this is a great opportunity for student presentations showing different approaches.

Group work (20 mins)
Student presentations (20 mins)

After each presentation, ask a few comprehension questions, calling on students at random to answer.

Record an effort grade for the lab.

Module 13 Producing Data for a Statistical Study

Learning Goals:

- Given a statistical research question, identify the population(s) and the variable(s).
- Categorize variables as quantitative or categorical.
- Distinguish between observational studies and experiments.

Estimated time: 45 minutes

This activity chunks concepts and provides *check your understanding* questions. Facilitate it using interactive discussion and/or iterative cycles of Think-Pair-Share.

Introduction and example (10 minutes)

Let students silently read the introduction or ask a student or two to read the introduction out loud to the class or just give a mini-lecture covering these ideas.

With the Big Picture, talk through the cycle and Think-Pair-Share for the example.

#1-2: Dissect a research question to identify population, variable, variable type (15 minutes)

Students have a surprising amount of difficulty with identifying the population and the variable, so you may want to fill in the 2nd row together. To help students dissect research questions, envision the data collection process, as follows:

- Who are we talking to? (These are the individuals who are part of the population.)
- What are we asking each individual? (Describe what information we collect from each person; this is the variable.)
- Sketch a spreadsheet to illustrate how the data would be recorded with each row representing an individual and the column defined by the variable.

#3-4 Observational studies vs. experiments (20 minutes)

Some of these questions do not have right or wrong answers, so students will need to present their reasoning. In situations like these, you might want to give students an additional opportunity to discuss their answers before the Share with the whole class. One way to do this is to use Think-Pair-Square-Share. After the Think-Pair segment, two pairs join together to discuss the answers. During Share with the class, randomly call on students.

Module 14 Observational Studies and Sampling

Learning Goals:

- Define statistical bias.
- Recognize biased sampling plans.
- Explain the purpose of random sampling in an observational study.

Estimated time: 35 minutes

This activity chunks concepts and provides *check your understanding* questions. Facilitate it using interactive discussion and/or iterative cycles of Think-Pair-Share.

<u>Introduction and #1</u> (10 minutes)

Let students silently read the introduction or ask a student or two to read the introduction out loud to the class.

Contrast the statistical meaning of “bias” with the everyday meaning of “bias.”

Think-Pair-Share on #1. During the Share, highlight the following ideas:

- The sample must represent the population. This is a problem in (1c) and (1d) because students who are texting in the Quad or using Twitter may text more often. These samples are likely to over-estimate the mean number of text messages sent by LMC students each day.
- Random samples can eliminate bias.
- Larger random samples are better than smaller random samples.

<u>Group work on #2</u> (25 minutes)

<u>Set-up</u>: (2 minutes)
#2 may be challenging for students, so you may want to set-up this problem with the entire class:

- Ask a student to read the introductory material above the graph out loud.
- Ask comprehension questions:
    - What are the 5^{th} grade students trying to estimate?
    - What does a dot represent in dotplot A? B? C?

<u>Group work</u> (10 minutes)

If groups finish early, have them write a short summary of the key points of this lesson.

<u>Ambassador Exchange</u> (8 minutes)

- After students work in groups on this problem, do an Ambassador Exchange. (4 minutes)
- Original groups then reconvene to discuss answers and make corrections if necessary. (4 minutes)

<u>Report out</u> (5 minutes) Since the concept of statistical bias can be difficult, have students report out answers and highlight the key ideas. This discussion will serve as the Closure for this lesson.

Key ideas for #2 discussion:

- A is the most biased method because it produces samples that are more likely to over-estimate the population mean weight. Students appear to have a bias toward larger snails.
- B and C are as likely to under- as over-estimate the population mean. These methods are not systematically favoring one outcome over another. Random selection eliminates the bias toward larger snails.
- C produces samples with less variability. Larger random samples vary less and thus do a better job of estimating the population mean.

SKIP Activity 14.2

Module 15 Collecting Data–Conducting an Experiment

Learning Goals:

- Identify confounding variables and identify features in experiment design that control the effects of confounding variables.
- Explain the purpose of random assignment in an experiment.

Estimated time: 35 minutes

<u>Discussion of the Introduction</u> (10 minutes)

- Give students about 3 minutes to read the introduction silently.
- Groups answer the following questions based on what they read:
  - How is an experiment different from an observational study?
  - What is a confounding variable?
  - What are two ways to control the effects of confounding variables?
  - How does random assignment control the effects of confounding variables?

SKIP #1 or do it at the end if you have time. This is a fairly sophisticated diagram.

<u>Group work on #2</u> (12 minutes)

<u>Ambassador Exchange</u> (8 minutes)

- After students work in groups on this problem, do an Ambassador Exchange.
- Original groups then reconvene to discuss answers and make corrections if necessary.

<u>Report out</u> (5 minutes) to answer lingering questions.

15.2 Unit 6 Lab

This lab uses an interesting real-life application to underscore the following ideas:

- Observational studies often do not control for confounding variables. For this reason, we do not draw cause-and-effect conclusions from observational studies.
- Confounding (aka lurking) variables may explain an association between the explanatory and response variables.
- Association is not causation.

We have allotted 40 minutes for class work on this lab. If you have been using a lot of group work and you also have students who seem unengaged, try individual work (15 minutes) followed by Speed-dating (20 minutes).

Discuss any lingering issues.

15.3 Unit 6 Group Project

Run the Unit 6 Project like a "conference style" poster session.

Estimated time: 60 minutes

Below are general notes for facilitating "Conference style" Poster Presentations. This is a similar set of instructions that we used for previous Group Projects.

Overview of how to run a "conference style" poster presentations

- Students work in groups on a difficult problem. Each group makes a poster to summarize their analysis and conclusion.
- One group member stays at the poster to present the findings. The rest of the group rotates from poster-to-poster to hear short presentations, similar to a gallery walk for a poster session at a conference.
- Students who are not presenting have a listening/responding protocol that fosters active engagement during each presentation.

What materials are needed?

- Blank notebook or printer paper.
- Poster paper (one to two sheets for each group of 4 students)
- Tape
- Markers (1-2 for each group of 4 students)
- Post-its (3x3in, two colors; for a class of 32, 168 post-its in each color)

<u>Set-up:</u> Approximately 5 minutes.

Review the instructions. Groups have to design both an observational study and an experiment in a short period of time, so encourage them to ask for help if they are getting bogged down.

<u>Group work</u>: Approximately 25 minutes.

Students work in groups of 4 to answer design an observational study and an experiment. Each group can choose the topic that interests them most or to save time, assign topics.

Monitor groups to ensure that they finish the design of the observational study and the experiment.

All students should be engaged in the making of the poster by writing up different sections on notebook paper and taping it to their poster.

Hang posters so that the four scenarios occur in a side-by-side sequence, followed by another sequence of 4 posters representing the 4 problems.

Poster presentations: Approximately 15 minutes

Dry run: 3 minutes for groups to practice prior to the gallery walk;
Gallery walk: four 3-min rounds

Instructions to students (same as always):

- One student will be the presenter. (The instructor can choose the presenter using random assignment or choose someone who looks disengaged or lost.).
- Groups have 3-5 minutes to do a dry run. This gives the presenter time to practice and get feedback from the group.
- Groups will rotate together (minus the presenter) when the lights flicker (or alarm sounds). Each round will last about 3 minutes.
- Before rotations begin, give stacks of the two colors of post-its to each presenter.
- Each student listening to a presentation will provide feedback to each presenter on post-its. One color is for a kudo: a positive note that captures something that was clear or compelling in the analysis. Another color is for a piece of constructive feedback: a note that highlights something that was unclear or perhaps incorrect or something that could be expanded in the analysis. Each student must leave two post-its (one of each type) at each poster.

Note: With 4 rotations, each presenter will do his/her presentation 4 times. This allows the presenters the opportunity to improve their understanding of their group's analysis and to improve their explanation in a low-stakes environment. This is a reason to pick a student who seems disengaged or lost.

Note: Insist that each student leaves both a kudo and a piece of constructive feedback at each poster. This helps students learn to mentally engage with a presentation, to treat their peers' ideas with respect, and to learn to disagree in positive and constructive ways.

Closure: Approximately 15 minutes.

(5 mins) Review feedback: Students return to home poster and discuss feedback. They can also fill in the presenter on what they learned from other posters.

(10 mins) Discussion

- Ask students to return to their seats for discussion.
- Ask students to reflect on the following issues:
    - Measuring the response variable in each scenario: Highlight how a few groups defined "best." How was the response variable measured in the observational study vs. the experiment?
    - Use of random selection and random assignment: which groups had interesting ways of dealing with random selection and/or random assignment?

Module 16 Introduction to Inference

IMPORTANT NOTE: The Module 16 activity was accidentally left out of the Math 110/110S packet. The page in the packet labeled Module 16 should be ignored.

Make copies of the Module 16 activity for your students (see next page) or just deliver a short lecture that communicates the ideas in the 16.1 Introduction, then project the Check Your Understanding item and discuss it.

Learning Goal: Recognize situations where statistical inference is, and is not, appropriate.

Estimated time: 10 minutes

This is a short activity on a very important concept. When students begin their end-of-course projects, they often write research questions that can be answered using descriptive statistics, e.g. who is the best home run hitter. This short activity highlights the difference between descriptive and inferential statistics.

If you copy and distribute 16.1, give students 3 minutes or so to quietly read the introduction or have several students share in the task of reading the introduction out loud to the class. Or if you don't make copies, just deliver a short lecture covering the ideas in the 16.1 Introduction.

Think-Pair-Share on *Check your understanding.*

Module 16.1 (MISSING FROM FALL 2019 MATH 110/110S PACKET)

Learning Goal: Recognize situations where statistical inference is, and is not, appropriate.

Introduction: We are now transitioning into the last part of the course: statistical inference. Let's briefly discuss the difference between descriptive statistics (the early part of this course) and inferential statistics.

Differences between Descriptive and Inferential Statistics

For descriptive statistics, we choose a group that we want to describe and then measure all subjects in that group. The statistical summary describes this group with complete certainty. For example, we can calculate the mean height of first graders at the local elementary school.

For inferential statistics, we define the population and then devise a sampling plan that produces a representative sample from the population. For example, we might want to estimate the mean height of first graders statewide by selecting a random sample of first graders from across the state.

A study using descriptive statistics is simpler to perform. However, if you need evidence that an effect or relationship between variables exists in an entire population rather than only your sample, you need to use inferential statistics.

With inferential statistics the statistical results incorporate the uncertainty that is inherent in using a sample to understand an entire population. Describing this uncertainty is where probability comes in. Because probability is involved, a statistical study involves data that comes from some random process, such as random sampling or random assignment.

In statistical inference, we will use probability density curves. These curves are mathematical models that represent the long-run behavior of random samples. With these models we will be able to describe and quantify the potential error in a random sample and the uncertainty we may have in the inference about a population that is based on a single random sample.

Check your understanding:

Which of the following studies involve statistical inference?

a) Who is the better homerun hitter: Barry Bonds or Babe Ruth?
b) Is the Contra Costa school bond measure expected to pass in the next election?
c) Is there a relationship between political party affiliation and views on gun control in the United States?
d) Is there a relationship between gender and views on vaping in our class?
e) What percentage of customers expect name brands to provide customer service on Facebook?

17.1 Understanding Sampling Variability: An Introduction to Sampling Distributions

Learning Goals:

- Use simulation and probability to draw conclusions based on data.
- Describe the sampling distribution for sample proportions and use it to identify unusual (and more common) sample results.

Total time: 50 minutes

Technology: Instructor needs access to the internet for StatCrunch and a random number generator.

Suggested set-up (5 minutes)

Big Picture Overview

Quick overview of course: Something like this … We began with analyzing data with graphs and numerical summaries. We then had a brief introduction to probability. In the last unit we talked about random sampling in observational studies and random assignment in experiments as a mechanism for reducing bias. In the last half of the course we will tie all of these strands together. We will now focus on the real work of the statistician: drawing conclusions about a population from a sample.

Project the Big Picture and talk through the death penalty example using the Big Picture. We provide talking points below.

At the end of April 2005, ABC News and the Washington Post conducted a poll to determine the percentage of U.S. adults who support the death penalty.

Steps in the statistical investigation:

1. *State a research question*

   Research question: What percentage of U.S. adults supports the death penalty?

   - Population: U.S. adults
   - Variable: Support the death penalty? (yes, no). Answers are the data.
   - Variable type: categorical, so we will summarize the data using a proportion.
     The population proportion and sample proportion describe the proportion in support of the death penalty.

2. *Produce Data:* We cannot poll all adults in the U.S., so we select a random sample to represent the population.

   This poll selected 1,082 U.S. adults at random. Each adult answered this question: "Do you favor or oppose the death penalty for a person convicted of murder?" Answers are the data.

3. *Explore the Data:* Summarize the sample data using graphs (pie charts, histograms, etc.) and/or numerical summaries (usually a mean or a proportion).

   In the sample from this poll, 65% favored the death penalty. This is a sample proportion of 0.65.

4. *Create a probability model:*

   Our goal is to determine the percentage of the U.S. adult population that supports the death penalty. We know that different random samples will give different results. A probability model will help us answer questions like these:

   - How much variability will there be?
   - What are the chances that a random sample reflects the opinions of the population within 5%?
   - How confident can we be in our conclusion about the population?

   These are the questions that are at the heart of statistical inference. If we want to be able to draw a conclusion about a population from a sample, we have to think about how accurate samples are. In other words, we need to understand how much random samples will vary.

5. *Answer the research question by drawing a conclusion about the population.*

The focus of today's lesson is on creating a probability model for sample proportions. Will the sample proportions have a normal shape? What is the mean and SD of the sample proportions?

Implementation (35 minutes)

This activity is mainly an interactive demonstration designed to help the class start thinking about how much random samples vary. The students will be answering questions related to the demonstration as you go through it.

Go over the **Introduction to this activity**, then begin the simulation.

Simulation (15 minutes)

- Assign each student a number. Randomly choose 10 students. Ask them to stand. Those with a cat raise their hand. Calculate the sample proportion that has a cat.
- Make a number line on the board from 0 to 1 with scale of 0.1, label the axis "sample proportions" and record the first sample proportion. Pick a student helper to come to the teacher's podium and open a StatCrunch spreadsheet to record the sample proportions. Pick another student helper to create a dot plot of the sample proportions on the board. You can also let one of these two students select the random samples. You will be busy orchestrating, determining the sample proportions and providing commentary on what is happening.
- Repeat. Reassure students that it is OK to be counted more than once. Record the second sample proportion. Repeat to collect about 20 samples.
- As the class generates the sampling distribution, students can reproduce it in #2.

Discussion of (3)-(10) (20 minutes)

After you conduct the simulation, use "think, pair, share" to involve students in answering questions (3) –(10).

For #4: Some students may still be struggling with the idea that the mean is a measure of "center" and the standard deviation is a measure of spread. Don't spend a lot of time here if they are struggling to estimate center and spread. The goal here is to get them to think about the values as sample proportions. Just get a few estimates and talk about those estimates relative to the sample proportions in the dotplot then move onto #5.

For #5: Project the StatCrunch spreadsheet so that students can see the list of sample proportions. Label the column "sample proportions." Use StatCrunch to find the mean and standard deviation.

Students may have trouble articulating what the mean and SD tell us. The mean is the average of the sample proportions and the SD is a way to measure how much the sample proportions vary about the mean.

After finding the mean and SD, use StatCrunch to make a dotplot and mark the mean and mean ± SD with vertical lines or do this on the dotplot that is on the board. This will help students think about the sample proportions as a distribution with a mean and SD. It will also prompt ways to identify typical sample proportions [loosely defined as those within one SD of the mean] and unlikely sample proportions [loosely defined as those more than two SD from the mean.]

#6: It is fine for students to eyeball an interval of typical values. They can also describe typical values as those within one SD from the mean.

#7: It is fine for students to eyeball "unlikely" values. They can also describe unlikely values as those more than two SD from the mean.

#8: The mean is our best estimate of the population proportion.

#9, be sure to make that point that when a statistician says that there is error in an estimate, this error is not due to a mistake. For a statistician, error describes how far off the sample proportion is from the population proportion. Therefore, the error in the sample proportions is expected and arises the variability we see in random samples.

Suggested Closure (10 mins)

Close with students in Think-Pair-Share for two tasks:

(1) write down the process for generating the distribution of sample proportions,

(2) describe how the distribution of sample proportions is related to the task of estimating a population proportion and estimating the typical amounts of error.

Have half of the class work on (1) and half on (2) in pairs. Share out to generate reasonable answers; you can encourage more precision by asking questions, such as "How were the samples chosen?" "Were all the samples the same size?", "What did we do with each sample?" (remembering that this is their first run at understanding a sampling distribution … which is tough!)

17.2 Sampling Distribution for a Population Proportion

Learning Goal: Describe the sampling distribution for sample proportions and use it to identify unusual (and more common) sample results.

Total time: about 60 minutes

Technology: Access to the internet for a StatCrunch demonstration

An overview for instructors:

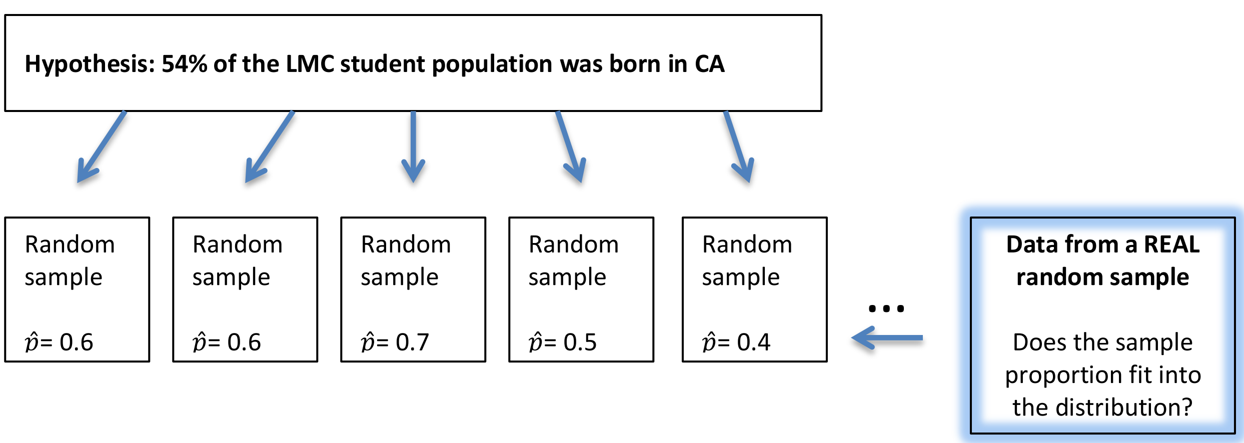
In 17.1 students examine a sampling distribution for the first time and see that the mean of the distribution is a good estimate for the population proportion.

In 17.2 we begin with a hypothesis about the population proportion and collect random samples from this population to examine the variability that occurs in random sampling. The activity ends with collecting a random sample from the class and thinking about whether the class sample could have come from the population with the hypothesized proportion. We will build on these ideas in subsequent activities in Module 17 to develop the logic of hypothesis testing.

Suggested set-up (5 mins)

Have students read the introduction to the activity silently or call on students to read out loud.

Afterwards, give a quick overview of the simulation using the context of testing a hypothesis about the proportion of the LMC student population who were born in CA. A diagram like the one below will be useful.



Suggested implementation (45 mins)

To reduce the cognitive load on students, the instructor is the only one manipulating StatCrunch in this simulation. This will help students focus on understanding this complex set of ideas.

- Open a StatCrunch spreadsheet. Under Applets, choose Sampling distributions. Click binary and explain to students that the simulation is binary because the variable is categorical with two values: Born in CA (yes/no)
- Enter p=0.54 and explain to students that you are setting the simulation to draw random samples from a population in which 54% are born in CA.
- Use the default Statistic(s), set to First: proportion. Click compute.
- Proceed through the simulation as indicated in the student activity.

As you conduct the simulation, give students time to think about the questions they have to answer in the activity. You may want to use a quick series of Think-Pair-Share as you go or just give 15 seconds or so of think time before calling on someone. Letting students ponder (and perhaps struggle) will produce deeper learning than students just writing down your answers to these questions.

The questions in the activity are repetitive. Don't worry about this; it is intentional. Call on different students to answer. Understanding sampling distributions is important and difficult. Don't rush through this. Give students time to process what is happening and why it makes sense.

Potential areas of difficulty

Here are a few questions that may be the most difficult for students to figure out:

For #4c, encourage students to describe the mean in context as the average of the sample proportions and the standard deviation as roughly the average amount that sample proportions deviate from the mean.

For #5c, the blue line marks values within one standard deviation of the mean. Thinking back to our early work with distributions of a quantitative variable, remind students that one way to identify typical values in a distribution is to use mean ± SD.

For #6c, the mean of the sample proportions should be close to 0.54. This makes sense because we expect most samples to have proportions that are close to 0.54. Over the long run, the sample proportions will average out to 0.54.

For #6d, the error is the deviation of a sample proportion from the population proportion. Since the population proportion is close to the mean of the sample proportions, the average error can be estimated with the standard deviation because the standard deviation is approximately the average distance of sample proportions from the mean.

For #6e, students may do this visually using the blue bar or calculate mean ± SD. Encourage both approaches by asking leading questions if it does not come up naturally.

For #7, you will need to select a random sample of 10 students from the class. Obviously, this is not a random sample from the population of interest, all LMC students. You might want to briefly note this but do not let this detract from the bigger ideas being developed here.

Closure (5 minutes)

If there is time, return to the initial diagram from the set-up to quickly review the simulation.

17.3 Effect of Sample Size on the Sampling Distribution

Learning Goal: Describe the sampling distribution for sample proportions and use it to identify unusual (and more common) sample results.

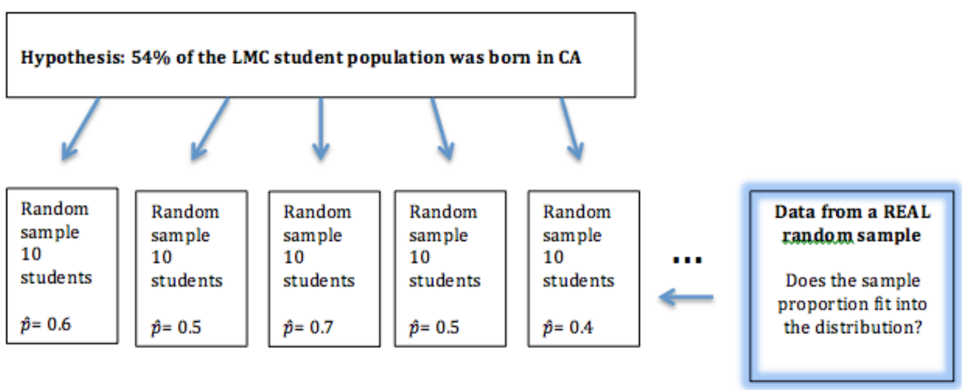
Learning Objective: Describe the effect of sample size on the distribution of sample proportions.

Estimated time: about 60 minutes

Suggested Set-up (15 mins)

Begin with a quick overview of the logic of hypothesis testing and it's relationship to the distribution of sample proportions by reviewing the following:

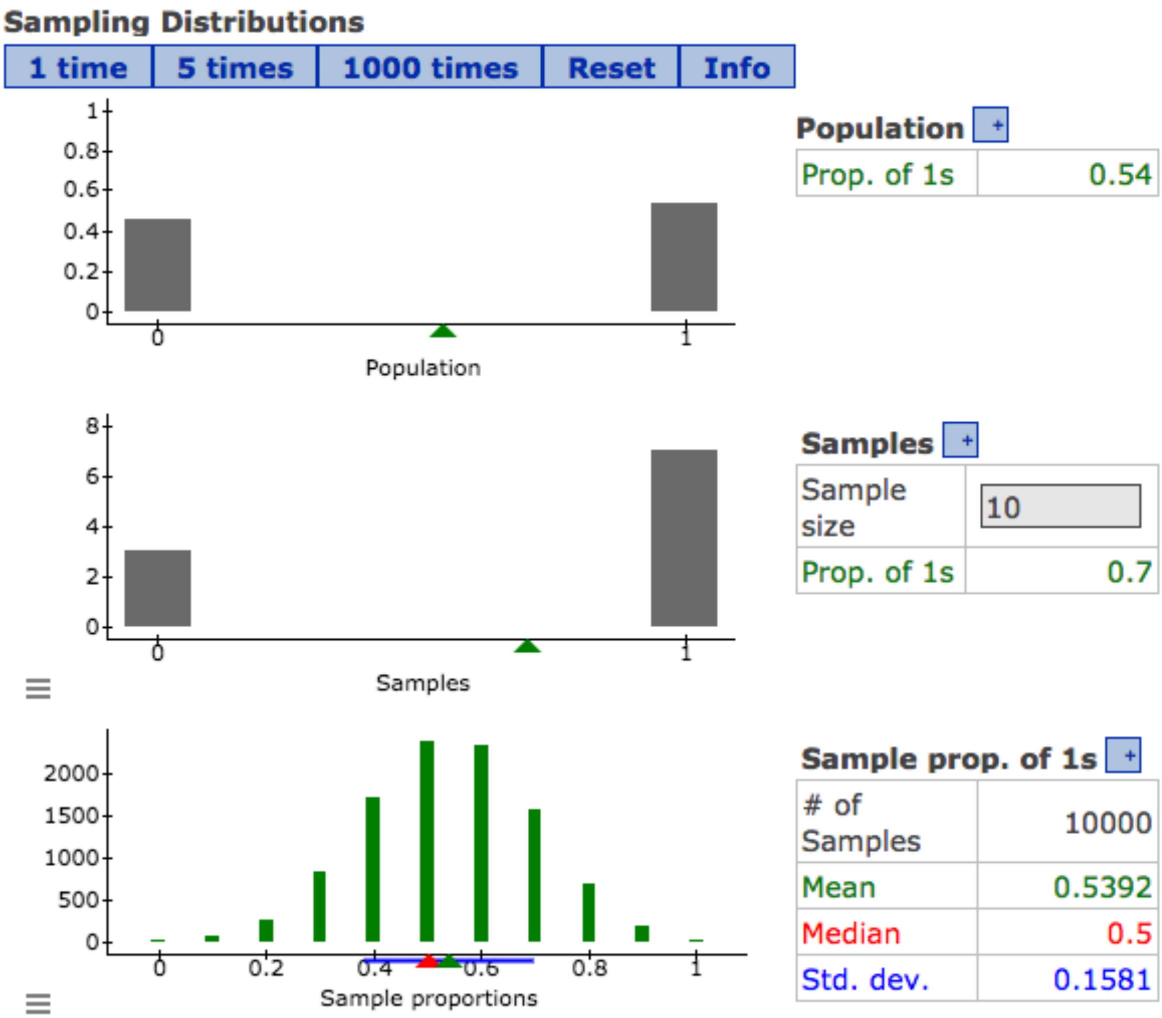
Project this diagram. Students also have this diagram in their activity. Even if you have just finished the previous activity, do this review. These are big, hard and fundamentally important ideas. You really can't err on the side of too much repetition here.



Talk through the following points to review:

- We began with a hypothesis about the population. Our hypothesis was that 54% of LMC students were born in California; written as a proportion, this is 0.54.
- We assumed this was true and simulated selecting random samples of 10 students. For each random sample we calculated a sample proportion that represented the proportion of the sample born in CA.
- We repeated this random sampling many, many times to generate a distribution of sample proportions.
- Finally, we collected a REAL random sample of students from our class and calculated the proportion born in California. We compared our sample to the distribution and thought about whether our hypothesis was responsible or not based on the REAL sample we collected.

Here is the same idea pictured in a StatCrunch simulation. Here we selected 10,000 random samples from a population with p=0.54. Each sample has 10 students in it.



Project this above image from a StatCrunch simulation and review these big ideas

- The shape of the distribution of sample proportions appears normal.
- We expect the sample proportions to be close to the population proportion. In fact, in the long run the sample proportions average out to the population proportion. In other words, the mean of the sample proportions is very close to the population proportion.
- We expect there to be some variability in sample proportions. The amount that a sample proportion varies from the population proportion is the error. The standard deviation of the sample proportions gives us an estimate of the average amount of error in the samples.
- The error is due to variability that occurs naturally in random sampling.

- Typical sample proportions fall within one standard deviation of the mean. Some unusual sample proportions are 0, 0.1, 0.2 and 0.9 and 1.0; these rarely occurred and were more than 2 standard deviations from the mean.
- We used a sample from our class to determine whether our hypothesis was reasonable. If the class' proportion was fairly close to 0.54, then we would conclude that our hypothesis was reasonable. If the class proportion was not close to 0.54, then we would conclude that our hypothesis is not reasonable. How we define "close" and "not close" depends on how much variability we expect in random sampling. This is way we need to run the simulation.

<u>Transition to Activity 17.3:</u> In the simulations we have conducted so far, we have always used a sample size of 10 students. In this activity, we are going to investigate what happens when we collect larger samples.

Have students Think-Pair-Share on question (1): Do you think that larger samples will be better? Why or why not?

Give students one-minute to think about this by themselves, then two-minutes to share their thoughts with a neighbor. Call on students to share their thoughts and their reasoning. No wrong answers here; all conjectures are fine.

Now repeat the simulation in StatCrunch for samples of 40, binary p=0.54. Do the simulation as you have done before, first with 1 sample, then another, then 1000 times, 2000 times. Narrate the simulation as you go to highlight major ideas …

- Our hypothesis is p=0.54;
- We select a random sample of 40 students.
- The sample proportion is xxx; we plot the sample proportion in the distribution.
- Repeat.
- This sample also has 40 students, but the sample proportion is different. This is not surprising because we expect random samples to vary.
- The sample proportions are not equal to 0.54 but, most of the time, they are close.

Suggested Implementation (about 30 mins)

Randomly assign students into groups to do (2)-(4). Address areas of difficulty as you circulate.

Possible areas of difficulty:

- For (2) students may ask why there are more bars in the distribution for n=40 than n=10. Ask them to run through some possible sample results with you and calculate sample proportions. [When n=40, the sample results can be 10/40=0.250, 11/40=0.275, 12/40=0.300 etc. The increments are 1/40=0.025. When n=10, samples results are in increments of 0.1, 5/10=0.5, 6/10=0.6, etc.]

- Students may confuse sample size with number of samples. Check for this confusion as you circulate. Ask, "how many samples are represented in the distribution?" and "how many students are in each sample." Remind them that we are investigating how the size of the samples affects the distribution of the sample proportions. Point out that in each simulation we collected 10,000 samples because we want to understand the long run behavior of sample proportions. This is why we collected thousands of samples.

- Students may have trouble when comparing the means and not see that the means are close to 0.54. You could ask them how the means relate to our hypothesis about the population proportion. Or you can encourage them to round to the hundredths place, then ask why the result makes sense [samples drawn from a population with p=0.54 should have sample proportions close to 0.54. In the long run we expect the sample proportions to average out to 0.54.]

- When students are comparing standard deviations, encourage them to also look at the sampling distributions. The line segment below the axis marks values within one SD of the mean, so half of the length of the line segment represents a SD.

- #3e: the phrase "within" can be confusing. Translate this as "average error that is no more than 10%" or average distance of a sample proportion from the population proportion is less than 10%.

- #3f: We want students to notice the left skew in the distributions for n=10 and n=40 and a normal shape for the larger size samples.

Suggested Closure (15 mins)

Go over #4. Ask students if they had the same answer for both (a) and (b). How can the answers be different when in both situations the error in the sample is 10%? To answer this question, investigate the issue of "close to" or "far from" the hypothesized p=0.80 in multiple ways:

- Visually by plotting 0.70 in both distributions and noting if it visually appears to be a typical value in the distribution.
- Calculating the interval mean±SD as an interval of typical values and seeing if 0.70 lies in that interval or seeing if 0.70 falls within the bar that denotes this interval.
- Calculating a z-score.

Think-Pair-Share: What are the big take-aways from this activity?

If you are pressed for time, just do the Think-Pair-Share on the big take-aways (5 minutes), and add (4) as a discussion item in Canvas.

17.4 Mathematical Model for the Distribution of Sample Proportions

Learning Goals:

- Use a mathematical model of the normal curve to represent the distribution of sample proportions for a given scenario.
- Use a z-score and the standard normal model to estimate probabilities of specified events.

Big Ideas:

- Statisticians use mathematical models of the normal curve to represent the distribution of sample proportions if appropriate conditions are met. The area under the normal curve represents probability.
- If conditions are not met, we use a simulation to examine the distribution of sample proportions. The relative frequency represents probability.

An overview for instructors:

- In 17.1 students examine a sampling distribution for the first time and see that the mean of the distribution is a good estimate for the population proportion and use the standard deviation to describe error in sample estimates.
- In 17.2 we began with a hypothesis about the population proportion and used a simulation to examine the variability that occurs in random sampling. We investigated whether a class sample could have come from a population with the hypothesized proportion.
- In 17.3 we used simulations to investigate the effect of sample size on the variability in sample proportions. We investigate how a sample proportion that deviates from the hypothesized population proportion by 10% can be typical when sample sizes are small and unusual when sample sizes are large.
- In 17.4 we model the distribution of sample proportions with a normal curve and use it to estimate probabilities. Unusual events have small associated probability.

Technology: Students will need access to the internet for this activity in order to use StatCrunch and Canvas. To save time, pull up StatCrunch and Canvas and sign in before class starts.

Total time: about 60 minutes

Suggested Set-up (25 minutes)

Spend about 10 minutes on the introductory material in the first page of the activity and about 15 minutes working through the examples. Plan to spend about 5 minutes on Example #1 and 10 minutes on #2.

<u>Introduction</u> (about 10 minutes)

- Have a student or two read the introduction out loud to get everyone focused.

- Do a quick Think-Pair-Share to fill in the blanks summarizing what we have already learned about the distribution of sample proportions.

- Provide a short rationale for moving away from simulations and to the normal model. Something like what is in the activity, "At a time when technology was less advanced (or non-existent!), statisticians were not able to run simulations. Instead, they developed mathematical models to describe the distribution of sample proportions. A mathematical model is an equation and its associated curve."

- Project the sampling distributions from the previous activity (reproduced for you convenience at the end of this set of notes) and note that for smaller sample sizes the distributions were skewed and for larger samples the distributions were more normal in shape. Then give the conditions for use of a normal model and explain that when these conditions are met, the normal curve does a pretty good job of estimating the behavior of sample proportions.
- Then quickly note that in the model the mean is p and standard deviation is $\sqrt{\frac{p(1-p)}{n}}$, where n is the sample size. You might also note that this formula for standard deviation shows that larger n will produce smaller standard deviation, as we observed in the simulations.

- Draw a normal curve and label the axis "sample proportions". Mark the mean and mean ± 1-standard-deviation. This should be familiar to students because of their previous work with the normal probability distribution for individual measurements.

- You don't need to say more than this. Work through the two examples to develop these ideas further.

<u>Examples </u>(about 15 minutes)

Spend about 5 minutes on #1 and about 10 minutes on #2

#1a: Show students how to check conditions using the formulas.
#1b: Use the formula to calculate the standard deviation as an estimate of the average amount of error in the sample proportions. [sd=0.0665]
#1c: Sketch the normal model with mean=0.67, sd~0.07. Label the axis "sample proportions." Note how this relates to the simulation.

#2a: Do a Think-Pair-Share to let students have a minute or two to think about this question. Call on students to answer with the goal of multiple ways to answer the question in order to reinforce connections. Bring up the following ideas if they do not surface in student answers:

- Mark the observed sample proportion of 0.62 on the simulated distribution and note that it falls on the blue line that is indicating typical sample proportions that are within one standard deviation of the mean.
- Mark 0.62 on the normal curve you sketched in #1c and note that it falls less than one sd below the mean.
- Calculate the z-score. It has been a while since students worked with z-scores, remind them that a z-score tells us how far a value is from the mean when we are counting in standard deviation units. So a z-score will tell us preciously how many standard deviations 0.62 is from 0.67. Write the z-score formula in words "$\frac{x\ minus\ mean}{standard\ deviation}$" and then fill in the values. Interpret the z-score [0.62 is about 0.75 (three quarters) of a standard deviation below the mean of 0.67]

#2b: Use the StatCrunch Normal Calculator to reproduce the image in the activity. You will probably not have time for students to do this with you, but they will get practice with this during group work.

Highlight the following:
- Where the values for the mean and sd come from
- How the question dictates the choice of "≤ 0.62".
- Area is probability (just like in Unit 15 when working with the normal curve.)
- A probability of nearly 25% means that a sample result of 62% or smaller is pretty common. This supports the thinking we did in #2a.

#2c: Use the OLI applet in Unit 7, Distribution of Sample Proportions (5 of 6), middle of the page, to reproduce the image in the activity and to talk through the answers. This is similar to what students did in OLI in Unit 5. You will probably not have time for students to do this with you, so just demo it.

#2d: Use StatCrunch to conduct the simulation described. As you set up the simulation, highlight the information given in #2d about what a coin toss represents, etc. Hit "1 run" a few times to illustrate what is happening, then hit "1,000 runs" 10 times to generate an additional 10,000 samples of 50-flips. Explain that the probability that a sample has 62% or fewer planning to vote YES is the number of times that this happens out of the total number of samples generated. In other words, it is a relative frequency.

Some students are probably going to be concerned that the probability estimates differ. Have a student read the final paragraph out loud and emphasize that all of our answers to probability questions will be based on models or simulations, which may give different but reasonable estimates.

Reassure students that you will not be grading based on a precise number but rather interpreting the probability and making a decision based on it.

Suggested Implementation of group work (20 minutes)

Randomly assign students to groups to work on (1)-(2).

There are lots of opportunities for productive struggle in these two problems. Plan to help students with technology (Instructions for the StatCrunch Normal Calculator and the Coin-Flipping Simulation are included in the activity). There will be opportunities for practicing these skills in the next module, too. So do not worry if students are struggling here.

Potential areas of difficulty:

#1a: Students may have difficulty identifying the value of the hypothesized population proportion in the statement “the majority of”. When circulating, ask them what a majority means in terms of a percentage, e.g. “if a majority of students prefer daytime classes, what percentage is this?” [at least 50%].

Suggested Closure (15 minutes)

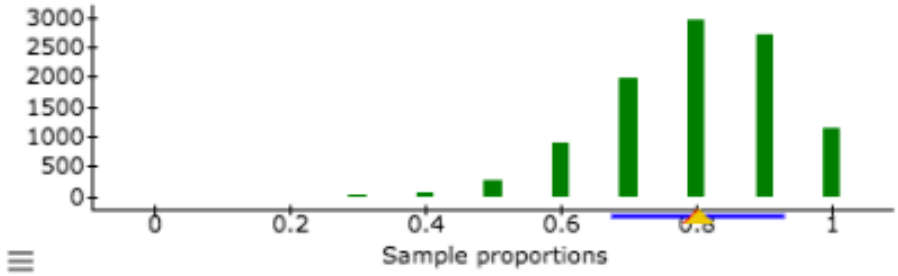
Student presentations: (15 minutes) Call on students to share their work using the document camera. They can also use StatCrunch at the teacher’s podium to recreate their work.

OR

Ambassador Exchange (10 minutes)
Ambassadors visit host groups and exchange ideas (7 minutes). Return to home group and discuss (3 minutes).

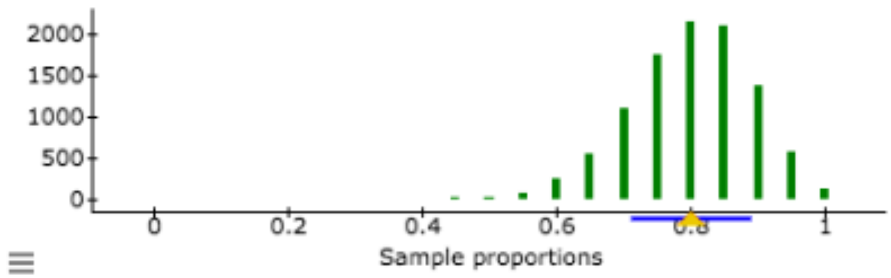
OR

Canvas discussion (0 minutes) Add a discussion item in Canvas as part of the homework. Randomly assign groups. Each student posts their responses to (1) and (2) and responds to two other student posts.



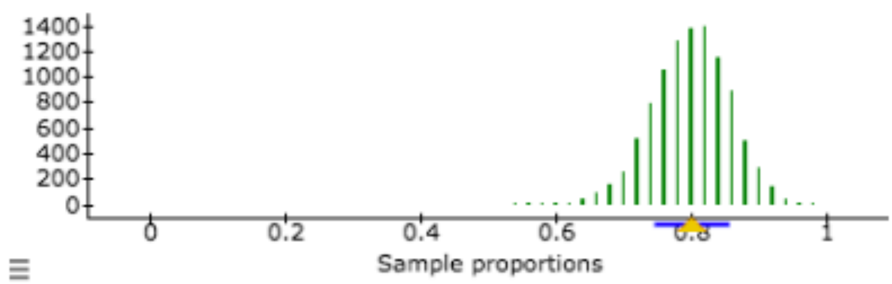
Sample prop. of 1s

# of Samples	10000
Mean	0.8022
Median	0.8
Std. dev.	0.1273



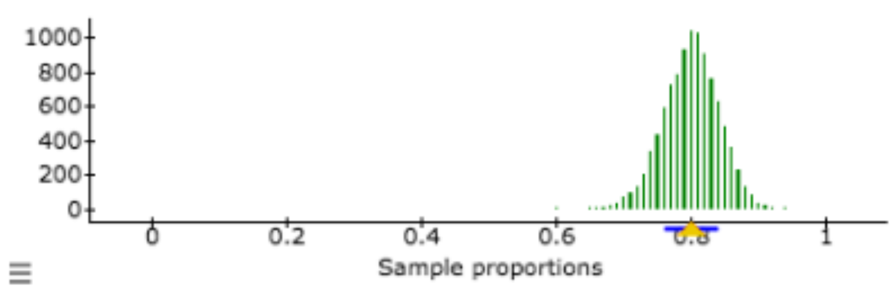
Sample prop. of 1s

# of Samples	10000
Mean	0.8002
Median	0.8
Std. dev.	0.0891



Sample prop. of 1s

# of Samples	10000
Mean	0.7995
Median	0.8
Std. dev.	0.0566



Sample prop. of 1s

# of Samples	10000
Mean	0.8001
Median	0.8
Std. dev.	0.0402

18.1 Introduction to confidence intervals

Learning Goal: Find a confidence interval to estimate a population proportion when conditions are met. Interpret the confidence interval in context.

Learning Objective: Use information from a media report of a survey to construct and interpret a confidence interval.

Estimated time: 50 minutes

Overview for the instructor:

This activity is the first of three activities about confidence intervals. In this introductory activity, students use poll results and a reported margin of error to construct a 95% confidence interval. They also practice interpreting confidence intervals. The formula for a confidence interval and the meaning of ‘95% confident’ come in subsequent activities.

Suggested Set-up (about 10 minutes)

Call on a student to read the introduction to the activity to help everyone get focused.

Work through the example in the activity.

Note: We will calculate the margin of error in the next activity. Here we are just using a reported margin of error and a sample proportion from a survey to construct a confidence interval.

Here are some talking points to use as you work through the example:

- Before answering (1), give students time to make sense of the graphic.
- When you calculate the confidence interval in (2), remind students that sample proportions will vary from sample to sample. Therefore, when we want to estimate a population proportion, we acknowledge this variability by adding and subtracting a margin of error. The result is an interval that describes a range of values that we think are reasonable estimates for the population proportion.
- In (3) the term “95% confident” will be investigated in depth later. For now, you might say that we cannot be certain that the population proportion lies in this interval. The phrase “95% confident” is a way of describing how trustworthy confidence intervals are.
- In (4) emphasize that all of statistical inference is based on random samples (or random assignment in the case of an experiment). Later we will develop additional conditions for using a confidence interval.

Suggested Implementation (about 25 minutes)

Randomly assign students to groups to work on #1-4.

Fast groups who finish early can prepare to present #2 or become Roving Tutors.

Potential areas of difficulty:
Students may struggle with reading the graph for (1). As you circulate, if groups are stuck here for too long, help them use the "employed full time" bar to complete the sentence given in the graphic: "% of mothers saying they spend ______ (too little, too much, about right) time with their children."

In (2) students may say, "yes we can conclude that the majority support a ban" because the sample proportion is 52%. Remind them that this is a sample proportion (a statistic), not a population proportion (a parameter). Ask, "what is the margin of error? Does the confidence interval suggest that that the population proportion is greater than 50%?"

In (3) students may incorrectly convert 0.08 percentage points to 8%.

Suggested Closure (about 15 minutes)

Students share answers to #2 using the document camera.

Discuss their answers to #4: What is the purpose of a confidence interval?

18.2 Finding a 95% Confidence Interval

Learning Goal: Find a confidence interval to estimate a population proportion when conditions are met. Interpret the confidence interval in context.

Specific Learning Objectives:

- Construct a 95% confidence interval from scratch.
- Check conditions that allow the use of a 95% confidence interval.

Total estimated time: about 45 minutes

Overview for instructors:
In the previous activity students constructed a confidence interval using a sample proportion and a margin of error stated in study that was reported in the media.

In this activity students learn to calculate the 95% confidence interval. In this activity we use p from a previous study to estimate the standard error. This is what we also did in Module 17. In Unit 8 students learn to estimate the standard error by replacing p with $\hat{p}$.

Also in this activity we do not dig into the meaning of 95% confidence. That happens in 18.3. We also do not introduce different levels of confidence. That happens in Unit 8.

Suggested Set-up (about 15 minutes)

Walk students through the introduction. The example gives you an opportunity to review several ideas:

- The conditions for use of the normal model were first introduced in Module 17. These conditions relate to the "IMPORTANT NOTE." Remind students that the distribution of sample proportions can have different shapes. We saw this in 17.3 when we investigated the effect of sample size on the shape of the sampling distribution. In order to say that we are '95% confident' that population proportion lies within the confidence interval, we must know that the normal model is a good fit for the sampling distribution.
- The standard error is a way to represent the average amount of error we expect to see in random samples drawn from the same population. Standard error is another name for standard deviation of the sample proportions.
- Error does not mean "mistake". Statisticians describe the amount that a sample deviates from the population proportion as the "error".
- The confidence interval is an interval of reasonable estimates for the population proportion.

Suggested Implementation (about 20 minutes)

Students work in randomly assigned groups on the two problems.

As you circulate, make sure students are choosing the correct percentage to estimate the standard error. In the two problems we intentionally reverse the order of the given information to make sure students are not just blindly using the first percentage as p in checking of normality conditions or in the standard error calculation.

Also carefully read the interpretations of the confidence interval. A good interpretation includes a description of the population proportion in the context of the problem. It is OK for students to use 18.1 problems as a template to interpret their interval.

Monitor for group difficulty with (2f) and (2g) and deal with this in closure if misunderstandings are prevalent here.

Suggested Closure (about 10 minutes)

Short assessment: Project one of the following problems using the document camera; choose the problem that addresses the most prevalent difficulty you saw during group work.

Do Think-Pair-Share.

1) According to the Department of Education's National Center for Education Statistics, 41% of graduating college seniors has credit card debt. Suppose that in a random sample of 100 California seniors, we find that 30% have credit card debt. Assuming that the variability in random samples will be the same in California as nationwide, estimate the proportion of California seniors statewide with credit card debt.

Which of the following is the correct calculation to find the confidence interval?

A) $0.30 \pm 2\sqrt{\frac{0.41(0.59)}{100}}$ B) $0.30 \pm 2\sqrt{\frac{0.30(0.70)}{100}}$

C) $0.41 \pm 2\sqrt{\frac{0.41(0.59)}{100}}$ D) $0.41 \pm 2\sqrt{\frac{0.30(0.70)}{100}}$

2) A Gallup poll reports that 48% of Americans drink soda daily. The methodology section states, "Results are based on telephone interviews conducted July 9-12, 2012 with a random sample of 1,014—adults, aged 18+, living in all 50 U.S. states and the District of Columbia. For results based on the total sample of national adults, one can say with 95% confidence that the margin of error is ±4 percentage points."
Which of the following interpretations is correct?

A) We are 95% confident that 48% of Americans drink soda daily.
B) We are 95% confident that 48% of 1,014 adults drink soda daily.
C) We are 95% confident that between 44% and 52% of Americans drink soda daily.
D) We are 95% confident that between about 40% and 56% of Americans drink soda daily.

18.3 What does "95% confident" really mean?

Learning Goal: Interpret the confidence level associated with a confidence interval.

Learning Objective: Explain the meaning of "95% confident."

Total time: about 50 minutes

Technology: Students need a way to take a random sample n=100 from N(0.8, 0.04). If they have access to StatCrunch, they can use the Sampling Distribution applet to do this.

Overview for the instructor: Problems (1)-(5) of this activity are a demonstration and class discussion. Problems (6)-(7) can be done as group work.

Suggested set-up (about 15 minutes)
Have a few students take turns reading the introduction and the paragraphs describing the simulation. Work through (1)-(5) as a class.

1) Students should be able to answer (1) on their own. Give a minute of think time, then call on someone to lead the class in checking the conditions for use of the normal model.

2) Have students open StatCrunch. To collect a random sample,

    - Open a blank spreadsheet (click Open StatCrunch).
    - Under Applets, choose Sampling Distribution.
    - Set to binary, p=0.8. Click "Compute."
    - Under Samples, set Sample Size to 100.
    - Click "1 time".

    Call on a few students to report their sample proportion. Note that there is variability as we expect.

    Use your sample proportion to demonstrate how to calculate the confidence interval.

    Write your interpretation. Suggested wording: "Of the population of community college students, we are 95% confident that between ____ and ____ intend to earn at least a bachelor's degree."

    Give students a few minutes to calculate their confidence intervals and write their interpretations. This will go quickly if they realize that the margin of error stays the same.

3) Project the normal curve using the document camera or draw one on the board. Plot 5 or so confidence intervals. To save time, call on students to give the lower and upper bounds of

their confidence intervals, instead of having students come to the board. Draw the intervals under the normal curve. Draw a vertical line through p=0.80 and down through the CIs so that it is clear which contain p=0.80.

For (5) ask students to raise their hands if they got an interval that did not contain p=0.80. Count how many to answer #5.

Ask for one sample proportion that gave such an interval. Quickly double check the calculation to verify that it did not contain p=0.80 and graph it.

Discuss why this happens. [It is not due to a mistake; some of the time p-hat is more than 2SE from 0.80. In the long run, what percentage of the time is p-hat more than 2SE from p?]

Suggested Implementation (about 20 minutes)

Students work in groups on (6)-(7).

Anticipate difficulty with (6g). We will deal with this closure. However, you can check their responses to (6d)-(6f). If these are correct, tell them that the answer to (6g) is a synthesis of their responses to (6d)-(6f). Another response might be: "95% confident" is a phrase that tells us how trustworthy our process is. What does the 95% refer to (95% of what)?

(7) will also probably be difficult. Let students grapple with this and deal with it in closure.

Suggested Closure (about 15 minutes)

Discuss (7), which is a very common misinterpretation of "95% confident". Here are the main points:

- "95% confident" is a short hand for "in the long run 95% of the time a confidence interval contains p."
- In general, we cannot say "there is a 95% chance that a specific interval contains p." It either does or it does not. But we can make a probability statement about what might happen before we collect the random sample. We can say "there is a 95% chance that the random sample will give a confidence interval that contains p."
- It is incorrect to say, "There is a 95% chance that p lies between two specific values" Why? because we cannot make a probability statement about an event that has already occurred. Once you have a specific interval, you cannot make a probability statement about it. For example, a pregnant woman has about a 50% chance of having a girl. But once she knows the sex of her baby, we cannot make a probability statement about the baby's sex. It does not make any sense to say "there is a 50% chance that you will have a girl" when we already know that the baby is a girl. Similarly, we cannot make a probability statement about a confidence interval that has already been calculated. If we look at the specific interval 0.77

to 0.93, does it contain p? [yes] We don't need to make a probability statement about this particular interval. We know it contains p.

- So why do we make the "95% confident" statement at all? Well, in practice we don't know if we have calculated an interval that contains p. It may or it may not. The best we can do is say that 95% of the intervals calculated in this same way (using a MOE equal to 2SE) will contain p. Unfortunately, we don't know if our interval is one of these, so we need to be able to make some kind of probability statement about the accuracy of the method we are using.

- What 95% confident means is that there is a 95% chance that the confidence interval associated with a randomly selected sample will contain p. This is the same as saying that there is a 95% chance that a sample proportion accurately estimates p within 2 SE.

One instructor reports that a student came up with the following analogy that is helpful in explaining "95% confidence": think of the confidence interval as a net. 95% of the time that you cast the net, it catches p.

Later, when we investigate different levels of confidence, you can think of a smaller net (lower level of confidence) or a bigger net (higher level of confidence.)

18.4 Introduction to a Hypothesis Test

Learning Goal: Test a hypothesis about a population proportion using a simulated sampling distribution or a normal model of the sampling distribution. State a conclusion in context.

Total estimated time: about 55 minutes

Overview for the instructor:

In the activities for Module 17 (Distribution of Sample Proportions), students were informally introduced to the logic of hypothesis testing using simulations as summarized below.

In this activity, we extend this informal introduction and talk about testing a claim. However, the formal mechanics and notation come in Unit 8. Therefore, we do not yet use Ho and Ha notation here. We do not yet use the vocabulary of a P-value.

Here is an overview of how the logic of hypothesis testing has developed to this point:

- In 17.2 we stated a hypothesis about a population proportion and examined the distribution of sample proportions when the hypothesis is true. We then gathered data from a sample and asked if that sample data could have come from the hypothesized population.
- In 17.3 we investigated the effect of sample size on the distribution of sample proportions given a hypothesized population proportion. This activity ends with a question that asks if a specified sample could have come from this population.
- In 17.4 students learned to use a normal model to make probability estimates in ways that foreshadowed the introduction of P-values (which happens in Unit 8). When the normal model was not a good fit, they used a simulation and relative frequencies.
- In 18.4 we use sampling distributions and relative frequencies to draw conclusions about a stated claim. In Unit 8 we will make the connection between relative frequencies and the P-value from the normal model.

Suggested set-up (about 30 minutes)

Walk the students through the example. Much of this is similar to what we did in Module 17, so you might want to use Think-Pair-Share for some questions to elicit student input.

Talk through the set-up for the simulations. Emphasize the following (or ask questions as you go to highlight the following):

- The probability of a head is set to 0.20. This represents our population proportion of 20% of community college freshmen owning credit cards.
- A "run" represents a random sample of 40 freshmen.
- The data collected has 25% owning a credit card.
- The simulation counts the number of samples with sample proportions >= 0.25.

(2a) Students can just eyeball it or calculate the standard error [~0.06] and note that 0.25 is less than one SE from 0.20.

(2b) Students used relative frequencies from a sampling distribution to estimate probabilities in 17.4. This is the same idea.

(2c) There may be some disagreement or confusion here. Some may argue that 25% is greater than 20%, so the sample data supports the claim. This is true; however, the evidence in support of the claim is weak. Why? Because the answers to (a) and (b) suggest that the sample comes from a population with p=0.20. The 5% error is not uncommon when the sample size is 40.

Proceed in a similar fashion for (3). We provided a clean graph in the appendix of the instructors' manual if you want to distribute clean copies to your class.

Talk through the summary.

(4) is just another way to visualize the points made in the summary.

Suggested Implementation (about 15 minutes)

Randomly assigned groups work on (5). Note that there is a problem with the image. The blue cloud should be a circle so that we can see the small number of samples with proportions ≤ 0.65. We provided a clean graph in the appendix of the instructors' manual if you want to distribute clean copies to your class.

In 15-minutes everyone may not complete (a)-(g). Due to time constraints, you may need to start the closure before everyone is finished. Hopefully, all groups will make it thorough part (e) before you start the short discussion of (5)

Suggested Closure (about 10 minutes)

- Ask by a raise of hands if cheating is less prevalent at the principal's school. Everyone will probably vote yes since 65% is less than 70%.

- Have students share their explanations. Here are some follow-up questions to ask to deepen the conversation:
    - If someone gives the explanation that 65% is less than 70%, ask is this strong evidence the cheating is less prevalent at this school or weak evidence? How do we know?
    - Ask, why we need to look at the sampling distribution before we can determine if 65% indicates that cheating is less prevalent at this school?
    - Elicit explanations that use both (c) and (d).

- If there is time, call on students to share their work on (f) and (g).

18.5 Unit 7 Lab

This is a challenging, but really interesting, lab. This lab involves core concepts for inference from Modules 17 and 18. If students can understand this lab, they have a good foundation for the rest of the course.

Estimated time: 45 mins

Students may not complete the lab in 45 minutes, but they will have an excellent start. Try to get them through (3c) so that you can check for understanding. The last question (4) is about confounding variables. It requires students to read an excerpt from another study. They may not have time to get this done in the allotted 45 minutes.

Facilitation suggestions:

Background material and videos (15 mins)

Give students time to read the introduction or have a few students read out loud to focus everyone. (12 mins)
Watch a few of the videos. NOTE: The link to the videos is broken. Here is the new link http://campuspress.yale.edu/infantlab/media/ (3 mins)

Simulation (5 mins)

Students can work in pairs to conduct the StatCrunch simulation or you can demo the simulation from the teacher's podium. The results of the simulation are not needed for the lab, but conducting the simulation helps students understand the StatCrunch print-out in the lab.

Group work (25 mins)

Randomly assign students to groups of 4. As you circulate, if you see or hear incorrect answers, point these out to students so that they can reconsider. Ask questions to steer them in the right direction if necessary.

Students will finish the lab for homework.

Areas of potential difficulty

#1: Students should be counting heads and reporting a sample proportion. Some may count tails and/or just report a count. (1b) a good sentence is something like … 62.5% of the random sample of 16 infants chose the helper toy.

#2b: A good label is something like … proportion of infants choosing the helper toy in random samples of 16 infants

#2c: The directions here do not make sense to some students. Ask questions like, how is the mean of the sample proportions related to the population proportion? What is the population proportion in this simulation? What is the formula for calculating the standard error?

#3b: We intended for students to use the relative frequency from the simulation (0.002), but some students may use the StatCrunch Normal Calculator based on their answers to (2c). Give kudos for creativity here then ask them if the conditions for use of the normal model are met here. Others may calculate the number of standard errors 14/16=0.875 is from 0.5, this works as a valid solution strategy regardless of sample size..

#3c: This is the logic of the hypothesis test. Students may still be shaky on this. Here are some questions you can ask if they are shaky here:

- What do the Yale researchers want to show about the proportion of infants choosing the helper? (That it is much greater than 0.5).
- Why is p=0.5 in the simulation? (The simulation helps us see what we can expect in random sampling if infants are arbitrarily picking the helper. This is why p=0.5, half the time we expect infants to choose the helper.)
- What are some sample proportions that would indicate that infants are arbitrarily choosing? (Sample proportions at or close to 0.5, e.g., 0.5, 9/16=0.5625, 7/16 = 0.4375, etc.)
- Does the observed sample proportion from the Yale study indicate that infants are arbitrarily choosing? How do we know?

Module 19 Estimating a Population Proportion

Learning Goal: Construct a confidence interval to estimate a population proportion when conditions are met. Interpret the confidence interval in context.

Specific Learning Objectives:

- Calculate confidence intervals using a sample proportion (instead of p from a previous study) to estimate the margin of error
- Calculate confidence intervals for different levels of confidence
- Describe the effect of increasing sample size on the margin of error

Total estimated time: about 80 minutes

Overview for the instructor:
In Module 18, students learned to calculate and interpret a 95% confidence interval using p from a previous study to check normality conditions and to find the standard error. In Module 19, p is replaced with p-hat.

Suggested Set-up/Introduction (about 45 minutes)

<u>Work through (1)-(6) and the summary with the class</u>

- (15 minutes) Questions (1)-(3) are a review, so you might want to do a series of quick Think (30 seconds)–Pair (30 seconds)–Share (1 min) to elicit student responses to each of the 6 questions in (1)-(3).

- (5 minutes) Questions (4) requires you to demonstrate how to adjust the normality check and the way we calculate the standard error using a sample proportion. This should go quickly because we are only tweaking a process that students are already familiar with.

- (25 minutes) Questions (5)-(6) and summary. Good opportunities for productive struggle here. Students will probably be able to figure out (5) and (6) on their own. Therefore, we suggest that you do a Think-Pair-Share on (5a), before computing the 99.7% confidence interval and finishing the problem. Similarly, let students work on (6) and share their estimates before moving to the summary where the critical z-score for the 90% confidence interval is given.

Implementation (about 20 minutes)
Randomly assigned groups work on (7)-(8).

Potential areas of difficulty:
The context of (7) and (8) may be confusing to students because the poll is about the public's confidence in various institutions (military, Supreme Court, etc.) and we are working with

confidence intervals. Obviously, “confidence” means two different things here. You may want to pre-empt this by pulling up the survey and noting the issue. Alternatively, let students struggle with it and monitor their responses to (7d) where the two uses of “confidence” are called out.

(7c): This is the first time we have had students compare two confidence intervals to determine if population parameters could be equal. If they do not think to look for overlap, ask questions like the following:

- In 2003 could the population proportion be 80%? Why or why not?
- In 2012 could the population proportion be 80%? Why or why not?
- How could we tell if there is an estimate for the population proportions that is the same in these two years?

(7d): Articulating the meaning of “90% confident” is always hard for students. Flag inaccurate interpretations and have students recall Activity 18.3 where we investigated the meaning of “95% confidence” in depth. Sketch a normal curve with confidence intervals as line segments underneath to remind them. Of course, pollsters use “confidence” to mean something like “trust”.

(8): Lots of conceptual questions here, but all should be within the students’ zone of proximal development at this point. In other words, if students are struggling here, ask questions to help them continue to think about margin of error and its relationship to confidence level. Make quick sketches of the normal distribution (or use the bell curve from (6)) and draw confidence intervals centered at p to show the relative lengths. Don’t rush to correct. You can work on this in closure.

(8d) There is no right answer to (8d). Hopefully, the tension between the desire for a high level of confidence and the desire for a small margin of error will emerge in students’ discussion of (8d). As you circulate, ask groups to explain their choice. Provide the other point of view and ask why they did not think it was as important.

Closure (about 15 minutes)

Discuss their responses to (8d). Here the discussion should elicit the tension between high levels of confidence and larger error.

Return to the counseling survey:

In an evaluation of academic counseling services on campus, the Chair of the Counseling Department plans to make changes in counseling services based on student feedback. She wants to be very confident in estimating the proportion of the student body that has had unsatisfactory counseling experiences. But she also wants to estimate this proportion within a reasonable amount of error.

What advice do you have for her? [High level of confidence with a large sample size to reduce the error.]

20.1 Hypothesis Testing

Learning Goals:

- For a claim about a population proportion, write null and alternative hypotheses.
- Recognize the logic behind a hypothesis test and how it relates to the P-value.
    - Compare P-values to a level of significance to draw a conclusion
    - State conclusions to hypothesis tests using the language of "reject the null" or "fail to reject the null" and statistical significance

Total time: about 80 minutes

Overview for the instructor:

In this activity we return to hypothesis testing. Previously, students practiced testing a claim by:

- using a normal model or a simulation based on a hypothesized population proportion,
- examining whether sample data was unusual
- finding the probability of a sample result exceeding the observed sample,
- using this information to decide if the evidence was strong enough to support the claim or whether the evidence suggested that the hypothesized population proportion was reasonable (difference between sample proportion and hypothesized population proportion could be explained by expected sampling variability)

Here we build on this and introduce students to the following vocabulary and concepts:

- Stating formal hypotheses using notation of Ho and Ha
- Significance level
- P-value
- Conclusion stated as "reject" or "fail to reject" Ho
- Statistical significance

Introduction/Set-up (about 40 minutes)

(5-7 minutes) Students silently read the introduction and discuss (1) in pairs for 3-minutes or so. Class discusses answers to (1).

(15 minutes) Walk through the example. This is essentially a lecture with students having the notes ahead of time in the form of a table. (Note: some graphs in this section should have circles instead of blue clouds. We reproduced these graphs and clean versions of these pages in the Appendix, in case you want to project or make copies for your students.)

We saw this same example in 18.4.

- To review, talk through the ideas in the first column of the table, top to bottom.

- Now talk through the table by rows. For each row, again review what we did before and then explain the associated vocabulary and concepts that are new (see list above). This will help students make connections.

  - Stating hypotheses: emphasize
    - the null is always “=”; alternative can be “>”, “<” or “≠”
    - We never make a hypothesis about a sample result because we can calculate it!
    - The hypotheses always have the same population parameter.
    - Verbalize the alternative hypothesis, this will help in writing a conclusion.

  - Assessing evidence: emphasize
    - The distribution of sample proportions is created assuming that the null hypothesis is true. In other words, we collect random samples from a population with the hypothesized proportion p.
    - The P in P-value stands for probability.
    - Don’t get confused by all of these different p’s. A lower case p represents the population proportion.

  - Stating a conclusion: emphasize
    - This is the same logic we have used before.
    - The reason we never “accept Ho” [we can never definitively say that p has a specific value. If the P-value is large, this indicates that the sample could have come from the population with hypothesized p. Therefore, we can only say that Ho might be true, at least we don’t have enough evidence to say it is false.]
    - Conclusions are stated in terms of our claim (Ha).

(10 minutes) Conclusions from a small P-value: Walk through the second table in a similar fashion. Review first top to bottom, then add in new concepts by talking across the rows to help students make connections.

(10 minutes) How small does the P-value have to be? Have students silently read the three paragraphs. Then go back and discuss how the conclusion in the 2nd table (n=300) remains the same if the significance level was 0.05 but changes if the significance level was 0.01

Implementation/Group work (about 30 minutes)
Randomly assign groups of 4. Monitor and clear up any issues by asking questions.

Closure (about 10 minutes)
Here are some ideas for closure:
Class discussion of (6): Students presentations using the document camera.
Ambassador exchange with a focus on (6). Clear up any lingering issues.

We will get a lot more practice with hypothesis testing in Module 21, so don’t worry if things are a bit bumpy at this point.

20.2 Type I and Type II error

Learning Goal: Recognize Type I and Type II errors.

Total time: about 10 minutes

Work through the 4 problems with the class. Use Think-Pair-Share on (3) and (4) to make it more interactive if you have time.

	We Reject H_0. (accept H_a)	We Fail to Reject H_0 (not enough evidence to accept H_a)
H_0 is true.	Type I Error	Correct Decision
H_0 is false. (H_a is true)	Correct Decision	Type II Error

If there is time, briefly discuss the following notes on probability of each type of error:

- If Ho is true, but we reject it, this is Type I error.

  With a 5% significance level, we will reject Ho when it is true, 5% of the time. With a 1% significance level, we will reject Ho when it is true, 1% of the time.
  In short, the probability of a Type I error is equal to the significance level.

- Type II error is harder to compute. If we attempt to decrease Type II error, Type I error will increase. Increasing the sample size will decrease Type II error.

Here is Michael Norris' summary:

- You can never have a Type I and Type II error at the same time.
- Type I means you are calling someone a liar, but you are wrong and they are telling the truth. (not good)
- Type II means someone is lying and you should have caught them but you missed it (very bad if the meat is spoiled.)

20.3 P-values and What They Mean

Learning Goal: Recognize the logic behind a hypothesis test and how it relates to the P-value.

Learning Objective: Interpret a P-value as a probability in the context of a statistical study.

Overview for the instructor: In Modules 17 and 18, we did informal work with simulations to develop the logic of hypothesis testing. In the previous activity, we added the formal notation and vocabulary of the hypothesis test, including the use of the term "P-value." In that activity students practiced drawing conclusions from the hypothesis test using the P-value.

In this activity we focus on interpreting the P-value as a probability. Our goal here is to understand the meaning of the P-value, not to use it to draw a conclusion.

Total estimated time: about 45 minutes

Suggested introduction/Set-up (about 20 minutes)

(5 minutes) Quickly talk through the example given in the introduction. This is a familiar context and a review.

(5 minutes) Give students two minutes to write the conclusion and compare with a neighbor. Then call on a few students to share.

(10 minutes) Give students a few minutes to read the 3 interpretations of P-value = 0.18. Then call on students during the class discussion of (1).

Suggested implementation/Group work (about 20 minutes)

As you circulate, flag incorrect answers and ask questions to nudge students in the right direction.

If groups finish quickly, let fast groups compare answers. Some can prepare to present #4 and others can be Roving Tutors.

Suggested closure (30 minutes)

<u>Discuss (4)</u> (10 mins)
Project student interpretations using the document camera and highlight how each interpretation contains the null hypothesis, a description of the samples included in the P-value, and the probability. To summarize, it might be helpful to sketch the sampling distribution as you slowly reread the interpretation of P-value, e.g. if the hypothesized null value is 0.49 [mark this on the normal model], then the probability that a sample proportion is 0.55 [mark this on the axis] is 0.03 [shade the area].

Snowball fight with a new problem (15-20 mins)
(3 mins) Project a new problem (suggestion below.) Quiet time for students to write their interpretations of the P-value.
(1 min) Wad up paper and 1, 2, 3 throw it!
(1 min) Find a snowball and return to your group.
(4 mins) Groups read snowball interpretations of the P-value and pick one that they think has a mistake and one that they think is accurate.
(5-10 mins) Share out a few good interpretations using document camera. Share out a few with mistakes and discuss what is missing or incorrect.

Suggested new problem:

A recent study suggests that 1 in 4 American adults have student loan debt. In a survey of our community, a random sample of 50 adults, 8 have student loan debt. Can we conclude that a lower proportion of our community have student loans debt?

In a test of hypotheses: H_o:p=0.25 and H_a:p<0.25, the P-value is 0.07.

Interpret the P-value as a probability statement.

21.1 Hypothesis Testing for a Population Proportion

Learning Goals:

- Recognize when a situation calls for testing a hypothesis about a population proportion.
- Conduct a hypothesis test for a population proportion. State a conclusion in context.

Total estimated time: **about 80 minutes**

Overview for the instructor:
In this activity we continue our work with hypothesis testing.

What students already know how to do:
- State formal hypotheses using notation of Ho and Ha
- Compare a P-value to a significance level to draw a conclusion
- State a conclusion as "reject" or "fail to reject" Ho
- State a conclusion using the phrase "statistically significant".

What is new to this activity:
- Two-sided tests
- Conducting an entire hypothesis test using the StatCrunch Normal Calculator or the OLI Normal Distribution Calculator

Suggested set-up (15-20 minutes)

<u>Introduction</u>

Have students read the introduction and then discuss the graphs. Here are the big ideas:
- P-value as an area corresponds to direction of the inequality in Ha. Why? If Ha is a "<" statement and if a sample proportion is less than the hypothesized p, then the sample supports Ha. But is the sample unusual? This requires us to calculate a probability based on the chance that a sample supports Ha AND is more extreme than what we observed, i.e. to the left of the observed sample proportion. Similarly for ">".
- P-value is double the area when Ha is a "≠". Why? Because we are only interested in how much the sample varies from the hypothesized proportion; we don't care if it is larger or smaller than po. Therefore, we want to calculate the probability that a sample is more extreme than what we observed, meaning further from po in either direction.

<u>Work through the example</u>

- In Step 1: Determine the hypotheses, ask students what word(s) suggest the "≠" in Ha.

- In Step 3 most of this should be familiar at this point to students. So you might want to give students a minute of think time then call on individuals to help with each part of Step 3.

- Have students shade in the left-hand area to show the two-tailed test.

- In this example, we fail to reject the null because the P-value is 2(0.035)>0.05. Give students time to write their own conclusions, then call on students to share their responses.

Implementation/Group work (about 45 minutes)

Potential areas of difficulty:

1a) Make sure students are writing a sentence to explain what p represents (p is the proportion of community college students in CA who "often" or "very often" come to class without completing readings or assignments.) The population is community college students in CA.

1b) Make sure students are using po=0.135 to check conditions. There may be some confusion about this because we use p-hat to check conditions for the confidence interval.

1c) Make sure students are using po=0.135 in the calculation of the standard error. We use p-hat in the calculation of standard error for confidence intervals, so this may be confusing. Anticipate some students to still be struggling with using a calculator to calculate SE and help them figure out how to do it.

1d) Make sure students are doubling 0.023 to find the P-value. A specified amount of decimal place accuracy is not important here but you may want to suggest rounding to 2 decimals places if they ask.

2) Students do not need to redo (1) with this smaller sample size to answer this question, but they can if they want to. The following type of answer is fine here:
"Yes, I think it might change because the SE will be much larger for samples of 100. Therefore, 10.5% will not be as many SE from 13.5%, which may mean that 10.5% is not unusual when sampling 100 students at a time."

If students are stuck here, ask them
- Which will change mean or Std. Dev. in the Normal Calculator when n is decreased from 500 to 100?
- How the Std. Dev. in the Normal Calculator will change? Will it get larger or smaller?
- What impact will a larger Std. Dev. have on the P-value?

3a) Again, make sure that students are clear about the population in their definition of p. Here p is the proportion of voters this year that are tattooed.

3c) Students may need some help with locating the x-value that marks the left-hand area. (0.225 is 0.025 from 0.2, so subtract 0.025 from 0.2.)

3d) Make sure students are looking at the StatCrunch printout P-value = 0.0481, not the normal curve. Hint for students: Think Empirical Rule.

3e) The sample is randomly selected !!!!!!!

4a) Here the alternative hypothesis is $p>0.26$. This is a bit tricky since p represents the proportion that has never done a class presentation. But the claim is class presentation is less prevalent.

OK, so the mantra is … make sure the population is clearly stated. A more precise definition of p is the proportion of students at a local community college who have never done a class presentation.

4c) Students may need a nudge to remember what SE represents. Recall that standard error is an estimate for the average amount of error we expect to see in random sampling.

4e) This is a one-sided test. So we do not need to double the area shown in the applet.

General observation: students may be confused because we are sometimes using the StatCrunch Normal Calculator and sometimes the OLI Z-score calculator and sometimes a StatCrunch simulation. Deal with this in closure.

Closure (15 minutes)

Ask students: When should we use the StatCrunch Normal Calculator and when should we use the OLI Normal Distribution applet?

The answer is (of course) either will work for us as long as conditions are met for use of the normal model.

Ask students: When should we use the StatCrunch Normal Calculator and when should we use a simulation such as the StatCrunch Coin-Flipping simulation?

The answer is: we can always use the StatCrunch Coin-Flipping simulation. Statisticians will use tools such as the StatCrunch Normal Calculator WHEN CONDITIONS ARE MET.

The truth of the matter is that the field of statistics is constantly developing new approaches to estimating probabilities. Z-scores and the standard normal curve were necessary when we did not have technology and we were using tables to estimate areas that represent probabilities. We had to translate everything into Z-scores so that we only had to deal with one normal distribution instead of a normal distribution for every possible mean and every possible standard deviation.

But now we have technology like StatCrunch. With the StatCrunch Normal Calculator, z-scores are really not necessary any more to find probabilities for a normal model. We can find the area probability if we know the mean and standard error.

Many statisticians no longer use the normal model. They just conduct a simulation with many, many samples and estimate the P-value using relative frequency. This is clean, simple, and accurate and does not involve the use of a mathematical model.

21.2 Cautionary Notes about Drawing Conclusions from a Hypothesis Test

Learning Goals:

- Distinguish statistical significance from practical importance.
- From a description of a study, evaluate whether the conclusion of a hypothesis test is reasonable.

Total time: about 60 minutes (or 120 minutes if you want to tackle the "They Would Say that Wouldn't They" article)

This activity may be challenging. There is nothing to calculate. Students have to think critically about what they have learned and use it to analyze the validity of a study. This is no small task but it is at the heart of statistical literacy skills.

Technically, there is no new content in this activity. However, there are plenty of opportunities to synthesize and apply previous concepts.

Suggested set-up (2 minutes)

Go over the information in the introduction that contrasts the unavoidable Type I and II errors with more serious problems that can arise in hypothesis testing.

Suggested implementation (25 minutes)

This activity is done in groups, punctuated with class discussion.

Here are two ideas for facilitating this activity:

- All groups work (1)-(4a–omit 4b) for about 25 minutes.
  OR
- Jig saw: Each group is assigned a problem and has 10 minutes to work on it. Reform groups of 4 so that each new group has an expert in one problem. Give groups about 15 minutes to teach each other.

Closure (30 minutes)

<u>Summarize as a class</u>: (15 mintues)
Students as a class to write a set of cautionary notes about statistical studies. Here are some examples about how the class might summarize these problems into a set of cautionary notes:

1) If a study does not involve a randomized controlled experiment, be cautious about drawing conclusions about how good a treatment is.

2) Statistical significance based on a pre-determined significance level may be misleading if researchers choose a one-tailed test over a two-tailed test. Statistically significant results are results that are common with large sample sizes. Always ask how big the difference is and weigh that against the cost and side effects of the treatment.

3) Similar to (2). Statistical significance does not necessarily translate into practical importance. Statistically significant results are results are common with large sample sizes. Always ask how big the difference is and weigh that against the cost of the intervention.

4) The main point here is that who sponsors a study may influence the reported results of the study. A secondary point is that research bias is itself the subject of research.

IF THERE IS TIME … (60 minutes)

Discuss the article *'They would say that, wouldn't they?' A reader's guide to author and sponsor biases in clinical research."* This is optional.

Set-up for discussion of article (10 minutes) Start by reading the introduction up to the title Financial Bias. Since the material is somewhat dense, you as the instructor should read it out loud. Pause periodically to paraphrase in an effort to demonstrate how you are thinking about what you are reading. As you think out loud, define words that may be difficult for students, such as ubiquitous.

Implementation (20 minutes) Each group will read and report on a segment of the article.

Suggested paragraph assignments are given below. If you have more than 6 groups, assign two groups to read the longer sections:

<u>Instructions to students:</u>

Have one person in the group read the first paragraph out loud. Then give everyone a minute or two to pick out a key sentence that they think captures the main idea. Compare and discuss the sentences that each person chose. Repeat with each paragraph your group is assigned.

Prepare a summary of your group's paragraphs in preparation for the class discussion.

Financial Bias
A. Paragraphs 1-3
B. Paragraphs 4-5

Financial Entanglements
C. Paragraphs 1-3
D. Paragraphs 4-7

E: Self Promotion Bias

F: ‘Eureka!’ Bias

Closure of discussion of article (30 minutes)

Groups present their summaries. After the presentations, give students 2-3 minutes to write out the main ideas that they are taking away from the discussion. Call on a few students to share their take-aways.

21.3 Unit 8 Lab

Technology: Access to StatCrunch required.

Estimated time: 50 mins

Students may not completely finish the lab in 50 minutes, but they will have a great start!

Students can do this assignment individually or in pairs.

(3 mins) If you are doing this lab before 21.4 Unit 8 Project, walk the class through *Instructions for accessing your data.*

Individual or Paired Work on #2 (20 minutes)

Check-in (10 minutes)
You might want to stop everyone after about 20 minutes and provide an opportunity for comparing answers to #2. Students will have different random samples, so this will be a rich conversation. Pair off (if students are working individually) or Square off to discuss in groups of 4 (if students are working in pairs.)

Individual or paired work on #3 and #4 (10 minutes)

Check-in (10 minutes)
Another Pair Off or Square Off to compare findings.

Assign an effort grade or let students revise their work and turn it in next class to be graded.

21.4 Unit 8 Group Project

Run the Unit 8 Project like a “conference style” poster session.

Estimated time: about 50-60 minutes

Technology: This project requires access to StatCrunch**.**

Below are general notes for facilitating “Conference style” Poster Presentations. This is a similar set of instructions that we used for previous Group Projects.

Overview of how to run a “conference style” poster presentations

- Students work in groups on a difficult problem.
- Each group makes a poster to summarize their analysis and conclusion.
- One group member stays at the poster to present the findings.
- The rest of the group rotates from poster-to-poster to hear short presentations, similar to a gallery walk for a poster session at a conference.
- Students who are not presenting have a listening/responding protocol that fosters active engagement during each presentation.

What materials are needed?

- StatCrunch access
- Blank notebook or printer paper.
- Poster paper (one to two sheets for each group of 4 students)
- Tape
- Markers (1-2 for each group of 4 students)
- Post-its (3x3in, two colors; for a class of 32, 168 post-its in each color)

Facilitation notes (with specific tips for this project)

Note: On page 237 in the student packet, in the poster instructions, under “For confidence intervals,” *remove the first bullet* (A sketch of the confidence interval appropriately located below the normal curve.). Why? Because we are not using p from the previous study in the calculation of the confidence interval. It does not make sense to assume that the sample proportions come from a normal distribution centered at p from a previous study.

<u>Set-up:</u> Approximately 10 minutes.

- Review the 8 options. Note that Options are paired around a single context. (Option 1&2 full-time students, Option 3&4 working college students, etc.) Assign each group a pair of options or just one, depending on the amount of time you can devote to this Group Project.
- Demonstrate how to select a random sample of StatCrunchU students.

- Demonstrate how to use StatCrunch to convert quantitative information into a categorical variable. Instructions are provided at the end of the project.

<u>Group work and poster construction</u>: Approximately 10-25 minutes depending on whether groups are doing both a hypothesis test and a confidence interval or just one of these.

Students work in groups of 4 to select a random sample of StatCrunchU students and use it to conduct a hypothesis test and/or construct a confidence interval.

All students should be engaged in the making of the poster by writing up different sections on notebook paper and taping it to their poster.

Hang posters so that the four scenarios occur in a side-by-side sequence, followed by another sequence of 4 posters representing the 4 problems.

<u>Poster presentations</u>: Approximately 15 minutes
Dry run: 3 minutes for groups to practice prior to the gallery walk;
Gallery walk: four 3-min rounds (rotating group gets to hear presentations on each of the 5 problems.)

Instructions to students (same as always):

- One student will be the presenter.
- Groups have 3 minutes to do a dry run. This gives the presenter time to practice and get feedback from the group.
- Groups will rotate together (minus the presenter) when the lights flicker (or alarm sounds). Each round will last about 3 minutes.
- Before rotations begin, give stacks of the two colors of post-its to each presenter.
- Each student listening to a presentation will provide feedback to each presenter on post-its. One color is for a kudo: a positive note that captures something that was clear or compelling in the analysis. Another color is for a piece of constructive feedback: a note that highlights something that was unclear or perhaps incorrect or something that could be expanded in the analysis. Each student must leave two post-its (one of each type) at each poster. Monitor this and intervene if a student is not following directions.

<u>Closure</u>: Approximately 15 minutes.

(5 mins) Review feedback: When the groups have rotated back to their own poster, they will naturally begin to look at the feedback on the post-its. They will usually also begin to explain what was on other posters to the presenter, who will probably be asking about this.

(10 mins) Discussion

- Ask students to return to their seats for discussion.
- Ask students what we have learned about the population of StatCrunchU students.

22.1 Introduction to the Distribution of Sample Means

Learning Goals:

- Describe the sampling distribution of sample means.
- Describe the effect of sample size on the variability of sample means.

Total estimated time: about 70 minutes

Before class:

Before class decide how you will have students select a random individual and a random sample from the 400 bottles.

Introduction/Set-up (40 minutes)

Introduction and #1 (10 minutes)

If you anticipate being pressed for time, skip #1, quickly go over the introduction and then move into the example.

If you have the time to do #1, ask students to silently read the introduction and answer (1), then compare answers with a neighbor and call on a few to report out. [b, c & e involve quantitative data]

- Quickly discuss each option highlighting the population of interest and the variable.

- If students are having difficulty, it may be helpful to think about the question we would ask individuals in the sample in order to gather the data; this question is the variable. For example, for (1a) we are asking community college students, “do you have a student loan?” (yes, no). For (1e) we are asking, “what is the interest rate on your student loan?” Students may use words like proportion, percentage or average to try to categorize the questions. This is a good idea but there are some options where they will need to think more carefully about the variable, e.g. (e) and (f). Another obvious “tell” is the word average. We summarize quantitative data with a mean.

Example and #2-5 (about 30 minutes)

Work through the example. You might want to do some Think-Pair-Share as you work through the example or just conduct an interactive class discussion.

Notes for discussing the example:

2a) Call on a few students to share their random individual bottle volume and locate a few in the distribution. The goal here is to help students understand that this is a distribution for individual bottles.

2b) A variety of responses are good here. Encourage multiple approaches.

- Visually, 11.9 is close to 12 in the distribution. It looks like a typical value.
- 11.9 is only one standard deviation below 12.
- Remind students that the z-score is $\frac{statistic - mean}{standard\ error}$, which is $\frac{11.9-12}{0.1}$= -1 in this case. Don't worry about using symbols like μ and σ; we will re-introduce those symbols in the next activity when we are developing the normal model.

2c) 67/400 = 0.1675, which makes sense because the distribution looks fairly normal and this is consistent with the Empirical Rule

2d) The quality control engineer will not be happy because there is about a 50% probability that the customer gets less soda than the advertised 12 fl.oz., which certainly would not be acceptable to consumer protection agencies or to customers. In industry, there is a 6-sigma rule that says the advertised amount should be 6 standard deviations below the mean to insure that customers are not cheated.

3a&b) Call on a few students to give their sample means. Plot a few in the dot plot and then locate a few more in the histogram. Again the goal here is to help students understand that they are now working with a distribution of sample means as opposed to a distribution of individuals in the population.

3c) A variety of responses are good here. Encourage multiple approaches.

- Visually, 11.9 is not close to 12 in the distribution. It looks like an unusual value.
- 11.9 is more than 2 standard deviations below 12, so it is unusual.
- Calculate the z-score: $z=\frac{11.9-12}{0.04}$= -2.5. (Again, do not worry about introducing formulas here. Just build off the idea that a z-score is $\frac{statistic - mean}{standard\ error}$ where here the statistics are sample means. In the next activity students learn that the standard deviation of sample means is $\frac{\sigma}{n}$, and that z = $\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$.)

4) Give students a minute to think about this. They should be able to guess that the distribution of mean volumes for 12-packs will be narrower. We have previously investigated the effect of sample size on the sampling distribution for proportions. In addition, it is intuitive to many students that larger samples will do a better job estimating a population parameter; therefore, there is less variability in larger samples.

5) From the perspective of the consumer and consumer protection groups, the new setting is better because the probability of getting a soda with less than the advertised 12 fl.oz. is 0. This

perspective would probably be shared by the quality control engineer who is trying to minimize the likelihood of customer complaints while also not giving away too much free soda.

Group work (about 20 minutes)

Students should not have much difficulty with the group work. Circulate to monitor their progress and intervene with questions if you observe misunderstandings.

They may need help interpreting "at least 79 inches" as 79-inches or greater.

Closure (10 minutes)

Run a simulation to investigate (4) from the group work.

- In StatCrunch choose Applets, Sampling distributions. In the pop-up window, set population to bell-shaped with mean 76.6 and SD 3.

- Choose one sample of size 15 to make sure students make the connection to what we did previously with measuring volume for a 6-pack of soda.

- Discuss shape, center and spread of the sampling distribution.

Then go over the class' responses to (5).

22.2 Modeling the Distribution of Sample Means

Learning Goals:

- Determine the mean and standard deviation of a distribution of sample means.
- Describe conditions necessary for use of the normal curve to model a distribution of sample means.
- Estimate probabilities using a normal model of the sampling distribution.

Total time: about 60 minutes

Technology: StatCrunch Normal Calculator required for one problem (4d) in this activity. You can also just do (4d) as a class during closure if students do not have access to technology.

Introduction/Set-up (about 20 minutes)

(5 minutes) Have students silently read the introduction. These ideas will be used in the example, so don't spend time going over the introduction in detail. The specific example will make more sense to them anyway and will provide a way for you to check their understanding.

(15 minutes) Work through the example interactively, giving students 10-30 seconds of think time before discussing each part. Alternatively, use Think-Pair-Share particularly on (a), (b) and (c).

Note: In general, body measurements in a large population can be modeled by a normal curve. If this is not stated explicitly and a problem is dealing with body measurements, it is safe to assume that a normal curve is a good model for the distribution of measurements in the population.

Note that (d) can be solved using the Empirical Rule. You may also want to quickly do a demo of this using the StatCrunch Normal Calculator and ask students what to enter for SD (100, not 500). Students will need to use the StatCrunch Normal Calculator for the last group work problem or you can have a student demo it at the podium if students do not have access to technology.

Group work (about 20 minutes)

Circulate while students work in groups. We do not anticipate that students will have too much difficulty with these problems.

If pressed for time, end group work early and do #4 interactively as a closure activity.

(2e) can be solved using the Empirical Rule, but if students want to use the StatCrunch Normal Calculator that is fine.

(4c) requires the StatCrunch Normal Calculator, though students will be able to say the difference is statistically significant without it because z=2.21. If they ask for a significance level (how great!), tell them to state their answer based on the significance level of their choice. For example, the sample results are statistically significant at the 5% level (P-value is approximately 0.01).

Closure (**about 15 minutes)**

Here are some ideas for closure. Choose one.

- Go over #4 with students sharing answers on the document camera.
- Let students work a new problem in a Think-Pair-Share format. Below is a problem you can use.

A McDonald's Quarter Pounder actually contains more than a quarter pound of beef, 4.25 ounces to be exact. But the cooked patty weighs around 3 ounces.

Suppose that the process for producing the uncooked patties has a standard deviation of 0.1 ounces. Quality control engineers assume that the production process produces a normal distribution of patty weights.

A quality control engineer weighs a random sample of 15 frozen uncooked beef patties for Quarter Pounders. She finds that the mean weight is 4.31 ounces.

In order to determine if the mean weight of this sample is usual, the food inspector will run a statistical analysis based on a model of the distribution of sample means.

1) What is the mean of the distribution of sample means for random samples of 15 patties?
   a) 1.10 ounces b) 3 ounces c) 4.25 ounces d) 4.31 ounces

2) What is the standard deviation of the distribution of sample means for samples of 15 patties?
   a) 0.007 ounces b) 0.026 ounces c) 0.1 ounces

3) What is the probability that samples of 15 burgers will have a mean weight of 4.31 ounces or less if the true mean weight of burgers is 4.25 ounces?

4) Is the observed sample's mean weight statistically significant? Why or why not?

23.1 Introduction to the Confidence Interval for Estimating a Population Mean

Learning Goals: Construct a confidence interval to estimate a population mean when conditions are met and a population standard deviation (σ) is not known. Interpret the confidence interval in context.

Estimated time: about 65 minutes

Suggested set up (about 10 minutes)

Introduction (2 mins)
Call on a student to read the two-sentence introduction. To check for comprehension, ask, What is the purpose of the confidence interval in this activity?

Discuss (1) and (2) (8 mins)

Use Think–Pair–Share to give students time to try to figure out the connections to previous material.

For #1: Think (1 min)–Pair (1 min)–Share (3 mins)

For #2: Think (1 min)–Pair (1 min)–Share (2 mins)
These are the same conditions for using a normal model to represent the sampling distribution of sample means. [The sample must be randomly selected. The distribution of the variable in the population has a normal shape OR the sample size is at least 30.]

Suggested implementation (about 40 minutes)

Randomly assign groups of 4 to work #3-6. These problems pull together a lot of course concepts. Monitor groups and press students to justify their answers and to write explanations. Note where students are struggling and use those problems in the Ambassador Exchange.

Suggested closure (about 15 minutes)

Ambassador Exchange: Chose problems based on your observations of groups.
Clear up lingering questions not addressed during the Ambassador Exchange.

23.2 Adjustments to the Confidence Interval for Estimating a Population Mean

Learning Goals:

- Construct a confidence interval to estimate a population mean when conditions are met and a population standard deviation (σ) is not known. Interpret the confidence interval in context.
- Explain the effect on the margin of error when we change sample size or confidence level.

Total estimated time: about 65 minutes

Introduction/Set-up (about 30 minutes)

Introduction (about 20 minutes)

As a class work through the introduction. Note that there are no questions to answer in the first two pages. There are no questions to answer until the Example.

Here are two ideas for discussing the introductory material:

- Do a mini-lecture and use the introductory material as lecture notes. Students can highlight or annotate the notes as you go through them.
  OR
- Paired reading activity: At this point in the semester, students should be able to read the introduction to the T-model with understanding; so instead of lecturing, you could do the following paired reading strategy:

  - (5 minutes) Quiet time to read and annotate the introduction and jot down questions that they have.
  - (5 minutes) Pairs discuss the main points and their questions with each other.
  - (5 minutes) Pairs answer to these questions:

    - How are the ideas in this activity similar to, and how are they different from, what we have done before?
    - When do we use a T-interval instead of a Z-interval to estimate a population mean?
  - (5 minutes) Students share their answers to the two questions above.

We anticipate that students may ask the following:

- Why is the degrees of freedom n-1?

  Don't get bogged down with this question. A thorough answer is too complex for our purposes here. Just say something along the lines of …

In the calculation of the sample standard deviation we use n-1 instead of n. This adjustment makes s a better estimator for σ. Therefore, the T-model also depends on n-1.

- Seems like we can use the T-model in situations where we would not be able to use the normal model. For example, when we aren't sure about the shape of the population's distribution, why do we now look at the sample's shape? We didn't do that for normal models.

  Don't get bogged down with this question. A thorough answer is too complex for our purposes here. Just say something along the lines of …

  For small sample sizes the T-model is flatter and fatter in the tails than the normal model; therefore, the 95% confidence T-interval is wider than the 95% confidence Z-interval. This means that the margin of error is larger with a T-interval for the same amount of confidence in our results. Because of this, we are willing to use T-interval for smaller samples if we can be reasonably sure that population's distribution can be modeled by a normal curve.

<u>Work through the example</u> (10 minutes)

This should be straight-forward and take less than 10 minutes. If you want to demo the T-score applet, it is in Canvas in Module 0 resources. This applet is used for two OLI exercises and one Checkpoint problem.

Implementation/Group work (about 25 minutes)

These problems are similar to those in a previous activity, so many students should do well here. This is another opportunity to drive home concepts that may still be muddy for some students.

Monitor group work and intervene with guiding questions if you see misunderstandings in student responses.

Closure (about 10 minutes)

Here are a few ideas. Choose one.
<u>Ambassador Exchange</u>: Pick problems with which groups struggled.
<u>Class discussion of key points</u>: Think-Pair-Share to develop a summary of key ideas. During Share, use a Whip Around, asking each group to share one key idea. Repeats OK if worded differently.

24.1 Hypothesis Test for a Population Mean

Learning Goals:

- Under appropriate conditions, conduct a hypothesis test about a population mean. State a conclusion in context.
- Interpret the P-value as a probability.

Technology: none. StatCrunch printouts are provided.

Estimated time: about 60 minutes

Suggested set-up (about 15 minutes)

At this point in the course, students will probably know the mechanics of a hypothesis test but they might be less clear about big ideas underlying a hypothesis test. Therefore, as you work through this example, use it as an opportunity to discuss big ideas again in a general way. More specifically, as you work through the example, consider discussing the following ideas. (Alternatively, you could save this more general discussion for closure and spend less time on the set-up.)

- Why do we need to do a hypothesis test?

A way you might explain this: We cannot just look at the difference between the hypothesized population mean and the sample mean and say "wow, looks small; guess what we assumed about the population's mean is reasonable." The sample results have to be judged against other random samples coming from a population where the null hypothesis is true. We have to look at the difference we see in the light of variability that occurs in these random samples.

- Why do we check conditions before running a hypothesis test?

A way you might explain this: A P-value is an area under a curve that is estimating how frequently samples with certain values are occurring. The curve is a mathematical model, in this case the T-curve. We need to verify that the mathematical model adequately describes the behavior of random samples before we use it to find a P-value.

- Why does a large P-value lead us to "fail to reject the null"?

A way you might explain this: When a sample result does not differ significantly from the hypothesized population parameter, then it is not unusual and the associated probability is large. Therefore, a large P-value tells us that we are seeing what we expect to see if the null is true. So we don't reject our null hypothesis.

- Why do we "fail to reject the null" instead of "accepting the null"?

A way you might explain this: The null hypothesis says the population has a specific mean. We can never be sure that the population has exactly this mean, but we can say that the evidence is not strong enough for us to doubt it.

Group work/Implementation (about 30 minutes)

Skip #5 if you are running out of time.

Potential areas of difficulty

1b) Students may not give precise definitions of μ or may misidentify the population. A precise definition will include information about the population and the variable. In this case, something like … μ is the mean rent paid for a 3+bedroom house in Antioch this year.

1d) Standard error is an estimate for the average amount that a sample mean will vary from the hypothesized $2500 in monthly rent. Specifically, on average the mean rent for a sample of 30 randomly selected houses in Antioch will vary about $54.74 from $2500.

1e) The T-stat tells us approximately how many standard errors her sample differs from $2500. Specifically, $2137.52 is about 6.6 standard errors below $2500.

2a) The first two options are correct.

2b) Option 2 is correct. If students are struggling with interpreting a P-value, try helping them deconstruct each statement by looking for the null hypothesis and for a statement about how likely certain sample means are.

3) The main issue here is that the students will not be able to run a T-test because the variable is categorical. Also, there is a problem with the sampling method because students studying in the math lab on a Tuesday afternoon may be less likely to work a lot at a job.

4) If necessary, prompt students to think about how many 4-minute songs to download (n>30 is best). The tricky part here is how to use random selection. With random selection we need to try to control for confounding variables that may impact download speed, such as location and perhaps time of day. Students may try to randomly sample 4-minute songs, which is less important because all of the songs have the same length.

Closure (about 15 minutes)

- Ambassador Exchange followed by class discussion of lingering issues.
- At the end of the lesson ask students to rate their understanding of each of the two learning goals using the metacognitive scale below.

1=not at all yet	**2=with a lot of support**	**3=with some support**	**4=with minimal support**	**5=on my own**

Ask students to raise their hand to quickly get a feel for the distribution of responses for each learning goal. Students are likely to still feel uncertain about interpreting P-values as probabilities. Reassure them that there is practice in Canvas for this.

24.2 Hypothesis Test for a Mean with Matched Pairs

Learning Goals:

- Under appropriate conditions, conduct a hypothesis test about a mean for a matched pairs design. State a conclusion in context.
- Interpret the P-value as a conditional probability.

Total estimated time: about 55 minutes

Overview for Instructor: This activity extends our work with hypothesis testing for a population mean to a matched pairs study design.

In a matched pairs design, the hypotheses are stated in terms of μ_d, the mean of the differences in the population.

The null hypothesis is always μ_d=0, which means that there is "no effect"; the differences average to 0.

The alternative hypothesis can be tricky because it requires us to think carefully about how the difference is defined and the desired effect. This is not highlighted in this example since the alternative is Ha: $\mu_d \neq 0$, but students will get practice with one-sided tests (and may struggle with determining whether to use > or <) in the group work.

T-procedures are robust which means that they can be used with smaller sample sizes. The conditions for use of the T-model in a hypothesis test are the same as for the confidence intervals, so nothing new here. In this example, conditions are met because the distribution of differences looks fairly normal in the sample, which suggests that a normal model can be used to describe the distribution in the population. Therefore, the sampling distribution can be modeled by a T-curve, and the small sample size is not an issue.

Suggested Set-up (about 15 minutes)

<u>Introduction and example</u>

This should go quickly because the logic of the hypothesis test is familiar at this point.

- Have a student read the introduction out loud to get everyone focused. Choose another student to read the post from allnurses.com.
- Work through the example.
- You may want to do (1a) from the group work together as a class, and use Think-Pair-Share on (1b) and (1c).

Here are the answers:

(1a) A lower cholesterol level is good, so we want "after" levels to be lower. This means that a positive difference "before minus after" shows that the drug is working. Therefore, the alternative hypothesis is μ_d>0.

(1b) A higher red blood cell count is good, so we want "after" levels to be higher. This means that a negative difference "before minus after" shows that the drug is working. Therefore, the alternative hypothesis is μ_d<0.

(1c) The claim is that cruise control increases reaction time, so we want "cruise control" reaction times to be higher. This means that a positive difference for "cruise control" minus "no cruise control" supports the claim. Therefore, the alternative hypothesis is μ_d>0.

Group work/Implementation (about 25 minutes)

Randomly assigned groups work on #2 and #3.

Potential areas of difficulty:

(2) Students may find it challenging to explain why these various conclusions are correct or incorrect.

Help them dissect their answers by

- checking that they understand that the evidence supports Ha
- recognizing Ha in the stated conclusions

If students are struggling here, this would be a good problem to discuss during closure. Here are the answers:

(2a) incorrect: We do not know that the 3-second difference holds for the population of all 4-minute songs. We only know that this is true in the sample.

(2b) correct: A small P-value means that the differences are statistically significant (not likely to be due to chance). However, the conclusion could be worded more strongly. The word "associated" could be replaced by more causal language since this is a randomized experiment.

(2c) incorrect: This conclusion is worded like a comparison of two distinct samples. We are not comparing one group who listened to classical music to another group who did not. This is a matched pairs design where every student experiences both treatments.

(2d) correct: A small P-value means that the differences are statistically significant (not likely to be due to chance). This conclusion uses causal language ("improved"), which is appropriate since this is a randomized experiment. This suggested generalization in the last phrase “which in turn suggests that listening to classical music may improve concentration” would be typical in a report for such a study.

Students may struggle with interpreting standard error and T-stat in (3)

(3c) The standard error of 20 pounds per acre is roughly the average amount of error (or deviation) we would expect to see in similar experiments if there really is no difference in the yield for the two types of seed.

(3d) The T-stat tells us that the differences observed in this experiment are approximately 1.69 standard errors above 0.

(3e) Maybe the most important question of the entire activity … when we observe a difference in the yields for the two seed types, how do we know if this difference is due to chance or due to difference in the seeds? A hypothesis test allows us to compare the difference we observe in the data to the expected differences that would occur in random samples from seeds that were identical.

Closure (about 15 minutes)

Ambassador exchange (8 minutes) followed by class discussion of (3e) for 7 minutes or so. Allow students to share their thoughts and add to each other’s explanations. Get other students to paraphrase the ideas in their own words.

25.1 Inference for a Difference in Population Means

If you are pressed for time, SKIP 25.1 and do the following:

(15 minutes) Quickly talk through the introductory material in 25.1 on the details of two-sample T-procedures highlighting connections with what we have done previously. Don't spend much time on this. We will use StatCrunch for all two-sample T-procedures, as is done in Canvas. You probably don't need to demo StatCrunch either. At this point in the course, students will be able to figure it out using the instructions in the activity.

(70 minutes) Let students work on the 25.3, the Unit 9 Project on Hypothesis Tests for Mean(s).

Total time for 25.1: about 90 minutes

Learning Goals:

- Under appropriate conditions, conduct a hypothesis test about a difference between two population means. State a conclusion in context.
- Construct a confidence interval to estimate a difference in two population means (when conditions are met). Interpret the confidence interval in context.

Overview: We will use StatCrunch to conduct two-sample T-procedures. Therefore, you do not need to emphasize formulas or T-curve applets for determining P-values for critical T-scores. This activity is structured like a project, but, instead of a gallery walk, all groups will present to the class.

Technology: Student access to StatCrunch and the internet is required for this activity. Put links to depression inventories some place where students can easily access them and not have to type the long urls.

Introduction/Set-up (about 15 minutes)

(5 minutes) Quickly talk through the introductory material on the details of two-sample T-procedures highlighting connections with what we have done previously. Don't spend much time on this. We will use StatCrunch for all two-sample T-procedures, as is done in OLI. Don't demo StatCrunch either. At this point in the course, students will be able to figure it out using the instructions in the activity.

(5 minutes) Give students time to silently read the article from BandarasNews.com or call on students to read it out loud.

(5 minutes) Have a student read the paragraph below the article out loud. Have another student read the information at the top of the 3rd page of the activity before the Options. Form groups. Assign each group an option.

Group work/Implementation (about 30-40 minutes)

1) Students may get really interested in the depression/anxiety surveys and start taking the survey and comparing results. Stop this when you see it happening. Have them go back and reread the instructions to (1). To build their summary, they might want to pick a few survey items to illustrate the types of questions used to calculate a score and give the overall scoring range.

2) Students may have difficulty interpreting the table. Monitor their responses to (2b) and guide as necessary. Encourage them to reread and discuss the example with Subject #1. Here are two examples of how to interpret a mean and SD:
   - Interpret the meaning of 7.27 [This is the average improvement for those in the treatment group as measured by the Hamilton scale.]
   - Interpret the meaning of 3.47. [This is an estimate for the average amount that improvement scores deviated from the mean improvement score for the treatment group based on the Hamilton scale. You could also say that this is an estimate for the average amount of error in the improvement scores.]

3) Let students choose their own hypotheses. You may need to help them determine the appropriate alternative hypothesis based how they defined Sample 1 vs. Sample 2. In other words, if Sample 1 is the treatment group, then Ha: $\mu1-\mu2>0$ fits the claim that the treatment had a positive effect (larger improvements). On the other hand, if Sample 1 is the control group, then Ha: $\mu1-\mu2<0$ fits the claim that the treatment had a positive effect (larger improvements). The actual researchers conducted two-tails tests.

   The StatCrunch results for each inventory is on the next page.

Closure (about 40 minutes)

Have each group present their results. (30 minutes) Then give students a few minutes to write their short overall summary for inclusion in the article from BanadarasNews.com. Have a few share-out. Praise the use of terms like “statistically significant”. (10 minutes)

Hamilton

Two sample T hypothesis test:
μ_1 : Mean of Population 1
μ_2 : Mean of Population 2
$\mu_1 - \mu_2$: Difference between two means
$H_0 : \mu_1 - \mu_2 = 0$
$H_A : \mu_1 - \mu_2 > 0$
(with pooled variances)

Hypothesis test results:

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
$\mu_1 - \mu_2$	3.67	1.2471434	28	2.9427249	0.0032

Two sample T confidence interval:
μ_1 : Mean of Population 1
μ_2 : Mean of Population 2
$\mu_1 - \mu_2$: Difference between two means
(with pooled variances)

95% confidence interval results:

Difference	Sample Diff.	Std. Err.	DF	L. Limit	U. Limit
$\mu_1 - \mu_2$	3.67	1.2471434	28	1.1153426	6.2246574

Beck

Two sample T hypothesis test:
μ_1 : Mean of Population 1
μ_2 : Mean of Population 2
$\mu_1 - \mu_2$: Difference between two means
$H_0 : \mu_1 - \mu_2 = 0$
$H_A : \mu_1 - \mu_2 > 0$
(with pooled variances)

Hypothesis test results:

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
$\mu_1 - \mu_2$	7.33	2.7136052	28	2.7012035	0.0058

Two sample T confidence interval:
μ_1 : Mean of Population 1
μ_2 : Mean of Population 2
$\mu_1 - \mu_2$: Difference between two means
(with pooled variances)

95% confidence interval results:

Difference	Sample Diff.	Std. Err.	DF	L. Limit	U. Limit
$\mu_1 - \mu_2$	7.33	2.7136052	28	1.7714317	12.888568

Zung

Two sample T hypothesis test:
μ_1 : Mean of Population 1
μ_2 : Mean of Population 2
$\mu_1 - \mu_2$: Difference between two means
$H_0 : \mu_1 - \mu_2 = 0$
$H_A : \mu_1 - \mu_2 > 0$
(with pooled variances)

Hypothesis test results:

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
$\mu_1 - \mu_2$	4.07	2.4049948	28	1.6923113	0.0508

Two sample T confidence interval:
μ_1 : Mean of Population 1
μ_2 : Mean of Population 2
$\mu_1 - \mu_2$: Difference between two means
(with pooled variances)

95% confidence interval results:

Difference	Sample Diff.	Std. Err.	DF	L. Limit	U. Limit
$\mu_1 - \mu_2$	4.07	2.4049948	28	-0.85640852	8.9964085

25.2 Unit 9 Lab

This is a challenging, but interesting, lab.

Estimated time: about 65 minutes

Suggested set up (about 3 minutes)

(2 mins) Call on a student to read the introductory material before the three excerpts.

(1 min) Assign each groups one question from the Questions to Answer (1)-(4).

Suggested implementation (about 30 minutes)

Groups read through the three excerpts, stopping after each excerpt to discuss the questions they have been assigned.

Groups develop an answer to their question and prepare to present it.

If groups finish quickly, have them compare answers with another group working on the same problem. Then have them begin work on a second question.

Suggested closure (about 30 minutes)

Groups present answers, probably two presentations per question given that two groups were probably assigned to each question.

25.3 Unit 9 Project

Total estimated time: about 45-70 minutes depending on your facilitation choices

Overview

This project is structured differently. Instead of working in groups, this project uses a cycle of individual work with StatCrunch, followed by speed-dating, and then time for individuals to revise their work and polish their conclusions. We will repeat this cycle three times with three separate problems.

Technology: StatCrunch is required for this activity

Materials:

- StatCrunch access
- A method for timing each part of the cycle and alerting students to proceed to the next part, e.g. using an internet clock with an alarm.
- A way to project the problems one at a time.
- Students will need some way to record their work for speed-dating, e.g. in a Word document or hand-written.

Set-up (about 15 minutes)

Instructions (5 minutes)

- Start with reviewing the three research questions are at the end of the activity. We will be using a random sample from StatCrunchU to answer each question.
- Show students how to find StatCrunchU and how to select a random sample of 100 StatCrunchU students. (Log into StatCrunch, go to **Resources**. Click on **StatCrunchU**. Set sample size to 100. Click **Survey**.)
- Go over the rest of the instructions in the activity to clarify what students will be doing. Note that the three research questions are at the end of the activity.

Warm-up (10 minutes)

Talk through the Warm-up.

- For (1) use Think-Pair-Share before calling on students to answer. (Answer: the only parts that are the same are StatCrunch procedure and hypotheses. Everything else depends on the random sample.)

- There is a lot to discuss in (2):
  - Walk through a demonstration of defining a categorical explanatory variable within the T=test screen. We are using Work (a quantitative variable that gives hours of work each

week) to define two groups: those who Work (Work > 0) and those who do not work (Work = 0). You can skip this if you plan to skip the first problem (Do StatCrunchU students who work have greater load debt than those who do not work?.)

  - Discuss why the alternative hypothesis is a "<" statement.
  - Now focus on the feedback. A better answer relates to the context of the problem, e.g. StatCrunchU students who Work take significantly fewer units than those who do not work.

Implementation (about 25-45 minutes)

<u>Three problem cycles with everyone doing all three problems</u> (about 45 minutes)

- Each problem cycle takes about 15 minutes.
- Instructor projects a problem on the overhead screen.
- (4 minutes) Students conduct the T-procedure and write a conclusion.
- (6 minutes) Two 3-minute rounds of speed-dating for students to discuss their work in pairs.
- (2 minutes) After speed-dating, students have 2-minutes to individually revise their work.
- Then the process will repeat for the next problem.

<u>IF YOU ARE PRESSED FOR TIME</u>, assign students to work on one of the three problems. (about 25 minutes)

- 1/3 of the class works on (1), 1/3 works on (2) and 1/3 works on (3).
- (4 minutes) Students conduct the T-procedure and write a conclusion.
- (6 minutes) Two 3-minute rounds of speed-dating among students with the same problem.
- (2 minutes) After speed-dating, students have 2-minutes to individually revise their work.
- (6-8 minutes) Two more rounds of speed-dating (3 minutes each), mixing problem types so that each student gets a chance to teach or be taught all three problems

StatCrunch notes:

For #1 and #2, use the **Stat, T-test, Two sample, with data**. Under Sample 1 and Sample 2, Values in: [choose the response variable].

  - For #1 under Where, type Work = 0 for Sample 1 and Work>0 for Sample 2.
  - For #2 under Where, type Gender = "Female" in Sample 1 and Gender = "Male" in Sample 2. Quotation marks and capitalization are required.

For #3, this is a one-sample T-test: **Stat, T-test, One sample, with data.**

Sample outputs:

For your convenience, here are the three questions, along with sample outputs for your reference (not to show students.)

3) Do StatCrunchU students who work have greater loan debt than those who do not work?

Sample output:

Two sample T hypothesis test:
μ_1 : Mean of Loans where Work>0
μ_2 : Mean of Loans where Work=0
$\mu_1 - \mu_2$: Difference between two means
$H_0 : \mu_1 - \mu_2 = 0$
$H_A : \mu_1 - \mu_2 > 0$
(with pooled variances)

Hypothesis test results:

Difference	Sample Diff.	Std. Err.	DF	T-Stat	P-value
$\mu_1 - \mu_2$	4796.6444	1073.9507	98	4.4663544	<0.0001

4) According to Debt.org, college students (undergraduates) have an average of $3,200 in credit card debt. (Source: https://www.debt.org/students/debt/) Do StatCrunchU students have less credit card debt on average?

Sample output:

One sample T hypothesis test:
μ : Mean of variable
$H_0 : \mu = 3200$
$H_A : \mu < 3200$

Hypothesis test results:

Variable	Sample Mean	Std. Err.	DF	T-Stat	P-value
CC Debt	2883.18	259.5936	99	-1.2204461	0.1126

5) Who has more credit card debt, female or male StatCrunchU students? How much more?

Sample output:

Two sample T confidence interval:
μ_1 : Mean of CC Debt where Gender=Female
μ_2 : Mean of CC Debt where Gender=Male
$\mu_1 - \mu_2$: Difference between two means
(with pooled variances)

95% confidence interval results:

Difference	Sample Diff.	Std. Err.	DF	L. Limit	U. Limit
$\mu_1 - \mu_2$	-1508.4166	521.18852	98	-2542.6982	-474.1349

Closure (about 5-10 minutes)

Students make up a research question that can be answered using the StatCrunchU survey data. For each question

- Identify the variable(s) and determine if they are quantitative or categorical.
- Determine the appropriate test and to identify situations for which we do not know the appropriate statistical test.
- Think-Pair-Share to use StatCrunch to answer the question (if there is time.)

If the question examines the relationship between two categorical variables, we will eventually use Chi-Square to answer it, e.g. Are StatCrunchU students who work more likely to have student loan debt?

If the question examines the relationship between two quantitative variables, we will not be able to answer it for all StatCrunchU students in an introductory course, e.g. What is the relationship between student loan debt and credit card debt for StatCrunchU students?

Module 26 ANOVA

Unit 10 is a single activity on the ANOVA One-Way F-test.

Learning Goal:
Conduct a hypothesis test for differences in three or more population means using an ANOVA F-test.

Total time: about 110 minutes

Technology: StatCrunch access needed in group work #3.

Suggested set-up (about 45 minutes)

As a class, walk through #1–#6 in an interactive way. You can use Think-Pair-Share on each problem, calling on students randomly to share out.

This will probably be relatively straight forward with the exception of #3. Students may have difficulty with inverse variation. (Larger variation within the sample gives smaller F.) Here are some questions to ask if they are stuck:

- One approach is to focus on the visual comparison more holistically. Which study looks like the samples could come from populations with the same mean? In other words, which set of samples have means with differences that could be attributed to sampling variability? (Think typical values here and note the overlapping boxes.) Which study looks like at least two of the samples come from populations with different means? (Look for boxes that do not overlap.)

- Another approach is to focus on the F-statistic. Which study will have a larger variation in sample means? (Answer: both have the same sample means and thus the same variation in sample means.)

  Which study will have a larger variation within each sample? (Answer: Study #1. Look at the spread in the middle half of the data, i.e. compare box lengths, and look at the standard deviations.)

  Which study will have a larger F-statistic? (This is the hard part for many students, i.e. understanding that when the denominator is larger, the F-statistic is smaller.)

Suggested Implementation (about 45 minutes)

Randomly assign groups to work on Group Work problems #1-#3.

As you circulate, check in with groups on the following:

#1 Can they articulate why conditions are met? Are they using SD and not SE in their conditions check? Do they understand that they do not need to be concerned with outliers when the sample is large (n>30)?

#2(b) Can students write the hypotheses in alternative ways?

#3 You might need to give some help with StatCrunch. Make sure students are checking conditions. With a sample size of 200, they will probably not need to make boxplots. If you have more time, #3 could be a poster gallery walk activity.

If groups finish early, here are somethings they can do:

- go back to #2b and state research questions and hypotheses for other hypothesis tests they have learned: for a population proportion, for a population mean, for a population mean using matched pairs.
- follow-up their hypothesis test in #3 with confidence intervals to get a sense of which class(es) of students account for the statistically significant differences.

Suggested Closure (about 20 minutes)

Here are some ideas for closure. You probably will only have time to do one or two.

- If groups had difficulty with #1 and/or #2, do a few rounds of speed-dating on one or both of these problems. Give students at least two speed dates on a problem. Speed-dating requires all students to talk about their work, which increases understanding and accountability. Multiple speed dates provide an opportunity for students who are less confident to get practice explaining concepts.

  After speed-dating, you can answer any remaining questions that were not resolved.

- Reshuffle groups and have them share and discuss their work on #3 (or do a poster session if you have time.) Alternatively, have some groups present their work on #3.

- Select a new random sample from StatCrunchU and have students tag team the execution of the ANOVA and the discussion of checking conditions, etc, at the podium.

- Do a Whole Class quiz with a new problem. The following data sets in the **Math 110 Los Medanos College** StatCrunch group can be used for ANOVA:

  - *Body Measurements:* Use Bin(age) as the explanatory variable. The data set is large and body measurements tend to have a normal distribution, so conditions will probably be met for any of the body measurements.

- *Cereal:* Use Shelf as the explanatory variable. All of the quantitative variables fail to meet conditions for the ANOVA due to smaller sample size and skew or outliers in the sample distributions.
- *Depression:* Use Treat as the explanatory variable and Time as the response. ANOVA conditions are met. This data set is used in Canvas practice opportunities.
- *Frustration:* Use Major as the explanatory variable and Frustration as the response. This data set is the leading example in the Canvas discussion of ANOVA. Data appears in both stacked and unstacked form if you want to discuss this with your students and show how to use ANOVA for each data format.; ANOVA instructions for both stacked and unstacked data are part of the Canvas Mod 26 discussion board.

Module 27 Chi-Square

Unit 11 is a single activity on the Chi-Square Test of Independence.

Learning Goal: Conduct a chi-square test for independence between two categorical variables in a contingency table.

Total time: about 100 minutes

Technology: StatCrunch access needed in group work #10.

Suggested set-up (about 55 minutes)

For each chunk of material, you can have a student read the section out loud and use Think-Pair-Share to check for understanding. If you feel it is necessary, ask some comprehension questions prior to student work on the *Check your understanding* questions.

Suggested time allotment:
Introduction and #1 (10 minutes)
State hypotheses and #2 (5 minutes)
Analyzing the two-way table–conditional distributions and #3 (10 minutes)
Chi-square test statistic and expected counts and #4 (15 minutes)
Conditions for the Chi-square test (5 minutes)
P-value and conclusion (10 minutes)

Suggested comprehension questions for each section:
#1: Do (1a) together asking these questions: What are the variables? If there are two variables, which is the explanatory variable and which is the response variable? Are the variables categorical or quantitative?

#2: students will probably be fine without additional comprehension questions.

#3: In the first row, calculate the percentage for the 1st cell together asking questions like these: Will each percentage in this row have 52 in the denominator? Why or why not?
In the first column, what does the 36% represent? [Answer: the percentage of child cereals on the bottom shelf.) What does 26% represent? Based on our work so far, are Target and Shelf independent? [Answer: no. Child cererals are much more likely to be on the bottom shelf compared to adult cereals.]

#4 We will not calculate the Chi-square test statistic by hand, so do not spend a lot of time on the formula. Simply point out that we will be determining how far the data (observed counts) differ from what we expect to happen when the variables are independent (i.e. the null hypothesis is true)–expected counts.

Do (4a) and (4b) together and Think-Pair-Share on (4c).

#5-#7 students will probably be fine without additional comprehension questions.

Suggested Implementation (35 minutes)

Randomly assigned groups work on #8-#10.

Areas of potential difficulty

#8: Students may jump into finding expected counts without noticing that the sample is not a random sample of all movies.

Suggested closure (10 minutes)

Ambassador exchange on #9 or #10, followed by discussion of lingering issues.

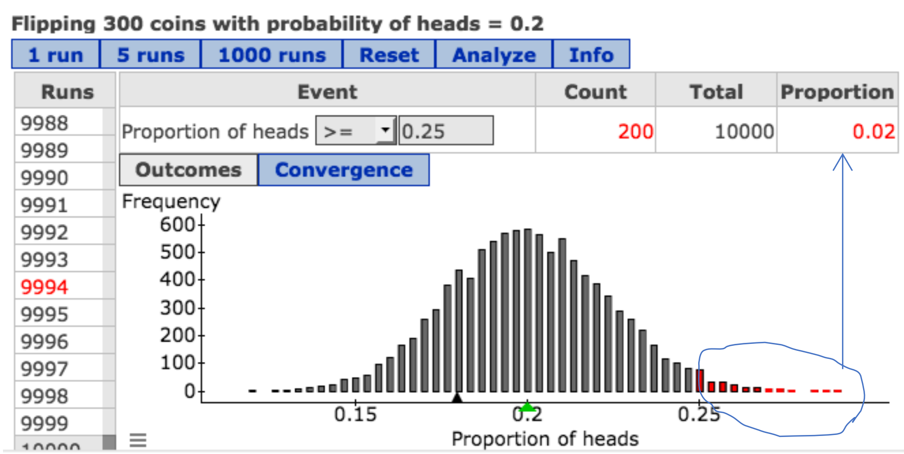
OR Speed date on #10 (2 rounds) if you feel that some students are disengaged or lost. Give students a few minutes to correct their work if needed.

APPENDIX

In version 1 of the Math 110 packet there were a few images that did not print correctly. The appendix contains revised pages. You can print these and distribute them to your class, or just project the corrected image.

From Activity 18.4 page 189

3) *Sample size of 300:* Here are the results from a coin-flipping simulation in StatCrunch with probability of a head set to 0.20. Each "run" is 300 coin flips representing a random sample of 300 community college freshmen.



a) With a sample of 300 freshmen, is a proportion of 0.25 unusual when the population proportion is 0.20? How do you know?

b) What is the probability that a random sample of 300 has 25% or more with a credit card? How do you know?

c) What does this suggest about our claim? Does our sample data provide strong evidence that more than 20% of the entire population own a credit card? Why or why not?

From Activity 18.4 page 191

Group work:

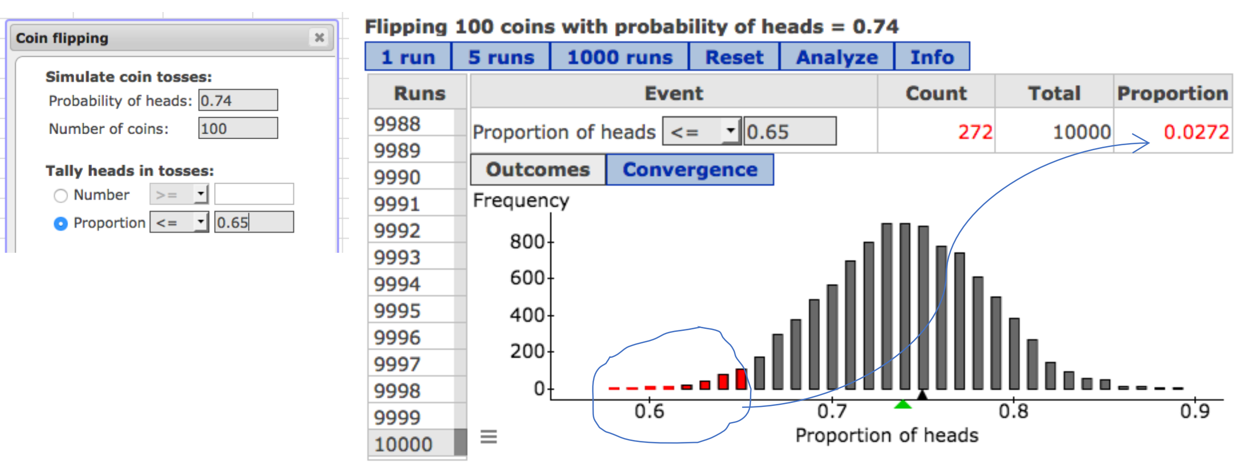
1) "Authoritative numbers are hard to come by, but according to a 2002 confidential survey of 12,000 high school students, 74 percent admitted cheating on an examination at least once in the past year."
Source: http://abcnews.go.com/Primetime/story?id=132376&page=1

A Principal of a local high school is appalled by this statistic. She wants to determine if the situation is any better at her school.

She hires a firm to conduct a confidential survey of a random sample of 100 students at her school. Sixty-five percent of the 100 students admit to cheating on exams.

a) Identify the following for this scenario:
   - The hypothesized population proportion
   - The sample proportion from the data
   - The sample size

b) The consultant runs a simulation and gets the following results. Explain how your answers in (a) relate to this simulation.

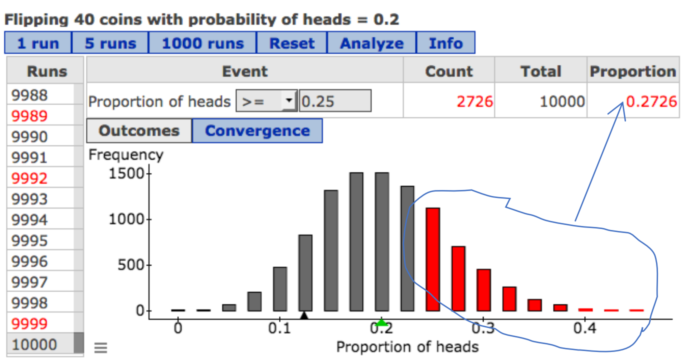


c) Is a sample proportion of 65% unusual when drawing random samples of 100 students from a population in which 74% cheat on exams? How do you know?

From Activity 20.1 page 208

<table>
<tr><th>What we did before</th><th>Notation and vocabulary of a hypothesis test</th></tr>
<tr><td>We start with the hypothesis that 20% of community college freshmen have a credit card this year. Our claim is that the percentage is higher.</td><td>Step 1: Determine the hypotheses.
Ho: p=0.20
Ha: p>0.20
p=proportion of community college freshmen with a credit card this year</td></tr>
<tr><td>To test the hypothesis, we select a random sample of community college freshmen. Suppose that the data shows that 25% of the sample has a credit card.</td><td>Step 2: Collect the data and report the sample results.
$\hat{p}$ = 0.25</td></tr>
<tr><td>Obviously, 25% is greater than 20%. But we need to determine if this 5 percentage point difference is typical or unusual when we look at samples coming from the population.

Here is the distribution of 10,000 sample proportions from samples of 40 community college freshmen.



Visually, we can see that 0.25 is not unusual in this distribution so the associated probability is large.

About 27% of the time, we expect a random sample to have 25% or more with a credit card when 20% of the population owns one.</td><td>Step 3: Assess the data.

Find the P-value.

The P-value is 0.27, which is large.

This indicates that the survey result of 25% is not unusual when sampling from a population where Ho is true.</td></tr>
<tr><td>Are we right that more than 20% of the all community college freshmen own a credit card? In other words, is our claim true?

Even though more than 20% of the sample had a credit card, the difference is not large enough to support our claim that the <u>population proportion</u> is also larger than 20%.

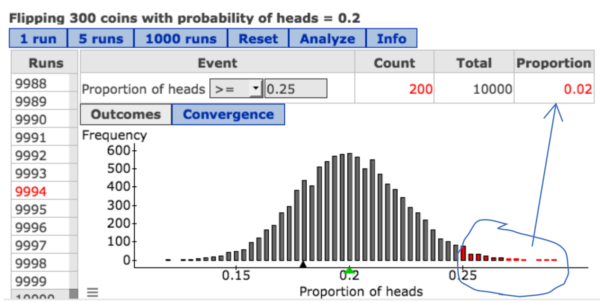
Therefore, we conclude that our claim is not true.</td><td>Step 4: State a conclusion
The sample evidence is not statistically significant. The observed difference between 0.25 and 0.20 can be attributed to sampling variability.

We do not have enough evidence to conclude that more than 20% of the population of community college freshmen owns a credit card.
Fail to reject Ho.</td></tr>
</table>

From Activity 20.1 page 209

You may be wondering … how does the conclusion change when the P-value is small?

In 18.4 we also ran a simulation of 10,000 samples of 300 students for this scenario. Let's *assess the data* and *state a conclusion* with these larger samples.

Visually, we can see that 0.25 is unusual in this distribution so the associated probability is small.  About 2% of the time, we expect a random sample to have 25% or more with a credit card when 20% of the population owns one.	*Step 3: Assess the data.* Find the P-value. The P-value is 0.02, which is small. This indicates that the survey result of 25% is unusual when sampling from a population where Ho is true.
Are we right that more than 20% of the all community college freshmen own a credit card? In other words, is our claim true? It is surprising to see random samples with 25% or more owning a credit card when 20% of the population owns one. In other words, this *difference is large enough* to support our claim that the population proportion is also larger than 20%. Therefore, we conclude that our *claim is true.*	*Step 4: State a conclusion* The sample evidence is statistically significant. The observed difference between 0.25 and 0.20 cannot be attributed to sampling variability. We have enough evidence to conclude that more than 20% of the population of community college freshmen owns a credit card. Reject Ho in favor of Ha.

You may be wondering … how small does a P-value have to be in order to accept the claim as true and reject the null hypothesis?

Since the hypothesis test asks us to answer "Is the claim true?" (yes or no), it is common practice to agree ahead of time on the definition of "unusual" sample results so that we can answer this question without any haggling. This definition of "unusual" is called a **significance level.**